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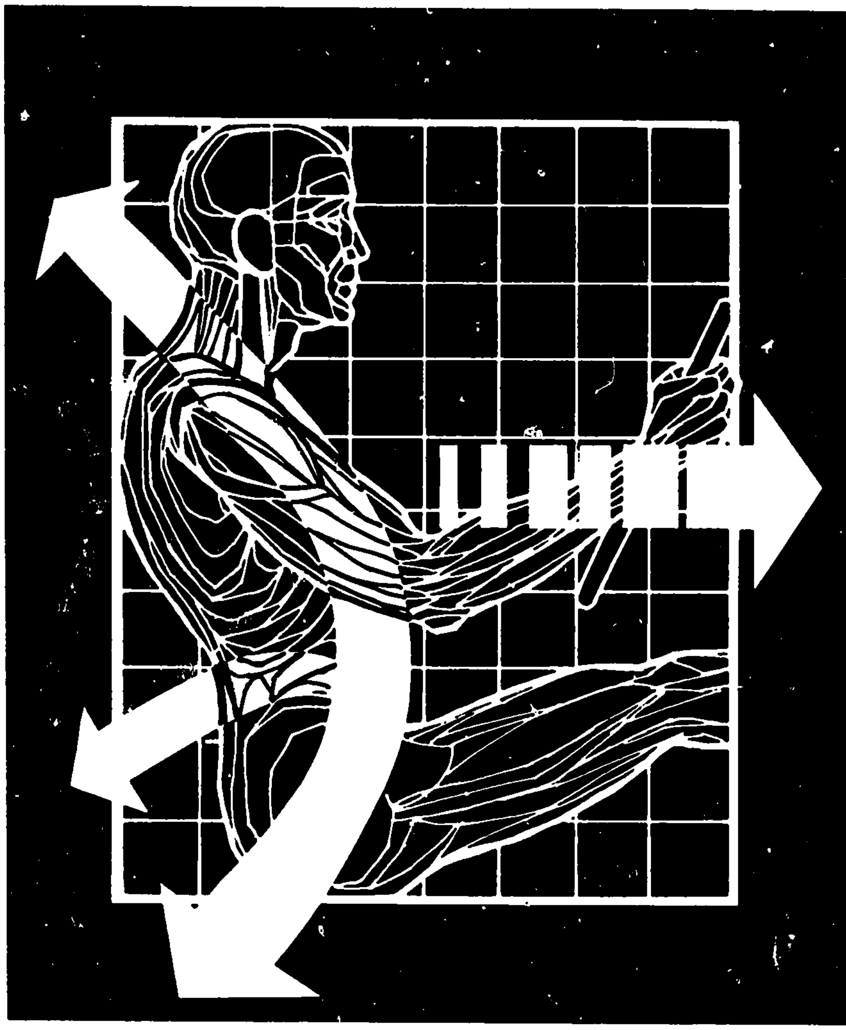
ABSTRACT

This collection of problems and experiments related to automobile safety belt usage is intended to serve as a supplement to a standard physics course. Its purpose is to convince the students that the use of safety belts to prevent injury or death is firmly supported by the considerations of physical quantities and laws which apply in a collision situation and hence that wearing belts while driving makes good sense. The material is divided into eight sections according to major physical concepts: velocity, acceleration, momentum, force, impulse, torque, energy, and stress and strain. Each section contains three to four classroom demonstrations, examples and problems, a laboratory experiment, and some programmed instructional materials. Examples and problems are meant to be convincing mathematical verifications that safety belts should be worn every time one drives. Answers to the problems and guiding comments are included. Laboratory exercises are intended to give the student an intuitive feeling for the relevant physical principles and their application to collision situations. Purpose, equipment, procedure, questions, and comments related to the experiment are enumerated. Finally, the programmed learning material contains concept definitions and problems (with answers) for purposes of review. (BL)

Physics and



Automobile Safety Belts

U.S. Department of Transportation • National Highway Traffic Safety Administration

SUPPLEMENTARY CURRICULUM MATERIAL:

Physics and Automobile Safety Belts

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INTRODUCTION

The materials in this book can perform a variety of services. The material is divided into sections according to major physical concepts – velocity, acceleration, momentum, force, impulse, torque, energy, and stress and strain. Each section may contain short (20 minutes or less) classroom demonstrations, examples and problems, a laboratory, and some programmed instructional materials. You may draw from these materials to design classroom or laboratory activities, to assign independent activity projects, to develop extra-credit or remedial assignments, or to construct quizzes and examinations. Some of the materials could be given directly to the student as review to help prepare for examinations. In short, these materials may be used in any way that suits your needs.

This collection of problems and experiments related to automobile safety belt usage is intended to serve as a supplement to a standard physics course. It's purpose is to convince the students that the use of safety belts to prevent injury or death is firmly supported by the considerations of physical quantities and laws which apply in a collision situation and hence that wearing belts while driving makes good sense.

The examples and problems are meant to be convincing mathematical demonstrations that safety belts should be worn everytime one drives. Example problems are labelled by E next to the problem number and are generally a more difficult type of problem. Almost every problem has some implication about safety belt usage, and appropriate comments have been included as guides. (It has been left to the instructor to point out and emphasize the important conclusions and relate the results of one problem to those of another.)

Laboratories are intended to give the student an intuitive feeling for the relevant physical principles and their application to collision situations.

Finally, the programmed learning material, containing concept definitions and graded review problems, is intended to reiterate the fact that wearing safety belt and harness while driving is good physical sense. This material can be used by the student as an exam review.

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VELOCITY

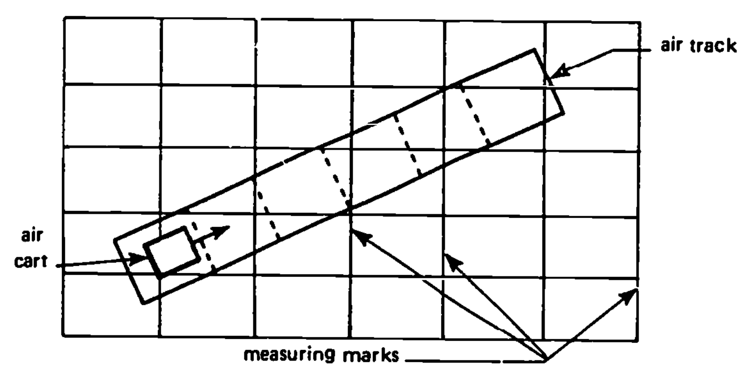
Demonstrations

Demonstration 1

Have students measure the time it takes a car to go a certain distance (change this distance starting with 20 ft.). This should help to build an awareness of how fast these speeds really are.

Demonstration 2

Demonstrate the vector property of velocity by finding the speed of an airtrack cart running parallel above a desk diagonal. Find the components of the cart's velocity parallel to the desk edges by using a large grid of 1 ft. squares and three stop watches (mark off the distance along the carts path also). Show that the velocity magnitude is related to the perpendicular components by the Pythagorean Theorem.



Demonstration 3

Measure some velocities using a CO_2 sled on a wire.

Demonstration 4

Run a set of velocity measurements using an airtrack and ask students to guess what the velocities are. Measure a number of high velocities for the car and then measure some slow velocities and ask students to guess the slower velocities. Expect to find guesses too low by a significant amount.

VELOCITY

Examples and Problems

1.E A man gets into his car, puts on his seat belt which takes 10 seconds, and drives the path shown below. He stops for 20 seconds at the first light, for 30 seconds at the second light and for 10 seconds at the third light. Due to traffic his average speed (while moving) along the (dotted) path shown is 10 mph. If each block is 600′ long:

a) How much time will his trip take?

b) What is his average speed? average velocity?

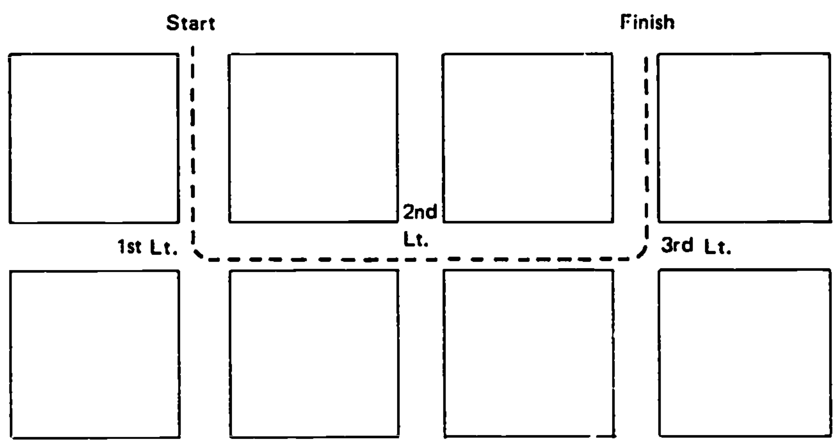
If he had not used a seat belt he would have eliminated roughly 10 seconds from his total transit time.

c) What would then have been his average speed?

d) What fraction of the total time is the 10 second buckle-up time?

e) Convert 60 mph to ft/sec

f) How long does it take to travel 15 ft. at 60 mph?



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Answer

a) T_T = total time

$= T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8$

T_1 = buckle-up time

T_2 = time from start to light 1

T_3 = time at light 1

T_4 = time from light 1 to light 2

T_5 = time at light 2

T_6 = time from light 2 to light 3

T_7 = time at light 3

T_8 = time from light 3 to finish

T_1 = 10 sec, T_3 = 20 sec, T_5 = 30 sec, T_7 = 10 sec

$T_2 = T_4 = T_6 = T_8$ = 40 sec

T_T = 230 sec

b) $S = d/T_T$

d = 2400 ft., T_T = 230 sec

S = 10.4 ft./sec

$$V_{AVE} = \frac{\Delta x}{\Delta t}$$

$\Delta t = T_T$ = 230 sec

Δx = 1200 ft

V_{AVE} = 5.2 ft/sec

c) $S = d/(T_T - T_1)$ = 10.9 ft/sec

d) T_1 / T_T = 1/23 or about 4%

e) 60 mph – 88 ft/sec

f) d = vT

d = 15 ft., v = 88 ft/sec

T = .17 sec

Comments:

Note how little time is necessary to insure against injury even for a *short* trip. For longer trips the percentage of total time used to buckle up becomes very small.

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2.E An automobile is passing a slower moving truck which is traveling at 20 mph. The car is initially 4 car lengths ($\approx 70'$) behind the truck. How much time does it take the car to close the gap if it travels at

a) 70 mph?

b) 50 mph?

c) 30 mph?

Answer

V_A = speed of the automobile
V_T = speed of the truck

Find the relative velocity which is $V_A - V_T$ since both are going in the same direction. Then find how long it takes to travel 70 ft. since this is their initial relative separation.

a) $T = d/(V_A - V_T)$
$d = 70$ ft., $V_A = 70$ mph, $V_T = 20$ mph
$T = 70/73$ sec $= .96$ sec

b) $T = 1.6$ sec

c) $T = 4.8$ sec

Comments:

Note how quickly a) and b) take place.

3. A car is moving down a street at 30 mph. A pedestrian suddenly steps out in front of the car and the driver is forced to swerve to the left by about 45°. Assuming that the car maintains its speed and that the driver wears a seat belt:

a) What change in velocity does he undergo? (Express your answer in terms of the original direction of motion.)

b) Which way would the driver tend to move if he were not wearing a seat belt?

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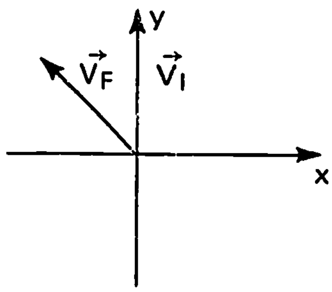
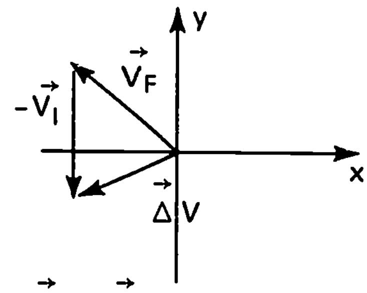
Answer

a) $\vec{\Delta V} = \vec{V}_{Final} - \vec{V}_{Initial}$

Original direction of motion is y direction





$\vec{\Delta V} = \vec{V}_F - \vec{V}_I$

$$\Delta V_x = -\frac{30}{\sqrt{2}} \text{ mph} = -21.2 \text{ mph}$$

$$\Delta V_y = (1 - \sqrt{2}) \frac{30}{\sqrt{2}} \text{ mph} = -8.8 \text{ mph}$$

*b) He would continue to move in the X direction only. Relative to the car he would appear to be pushed to the right across the seat.

Comments:

*The seat belt assures the man that he is not thrown from behind the wheel, hence substantially improving his future control of the car.

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Comments:

Note how quickly a car moving at 60 mph goes the length of a room.

4. An unrestrained passenger is riding in the front seat of a car traveling at 60 mph. The driver is forced to stop abruptly. The passenger, originally at rest in the car, suddenly finds himself moving towards the dashboard at 20 mph. If the dashboard is two feet away from the man and his reaction time is one tenth of one second, can the man get his arms up in time to protect himself?

Answer

Compare the time, T, it takes to move 2 ft. at 20 mph with 0.1 second.

$T = d/v = 0.07$ sec

The man can not react in time to protect himself.

Comments:

In a collision situation at 60 mph an unrestrained passenger will be moving towards the dash board at a speed of almost 60 mph.

5. Find the velocity of an unrestrained passenger in an automobile 0.06 second after initial contact in a 30 mph head on collision. The displacement of the passenger relative to the car is given as a function of time in the accompanying graph.

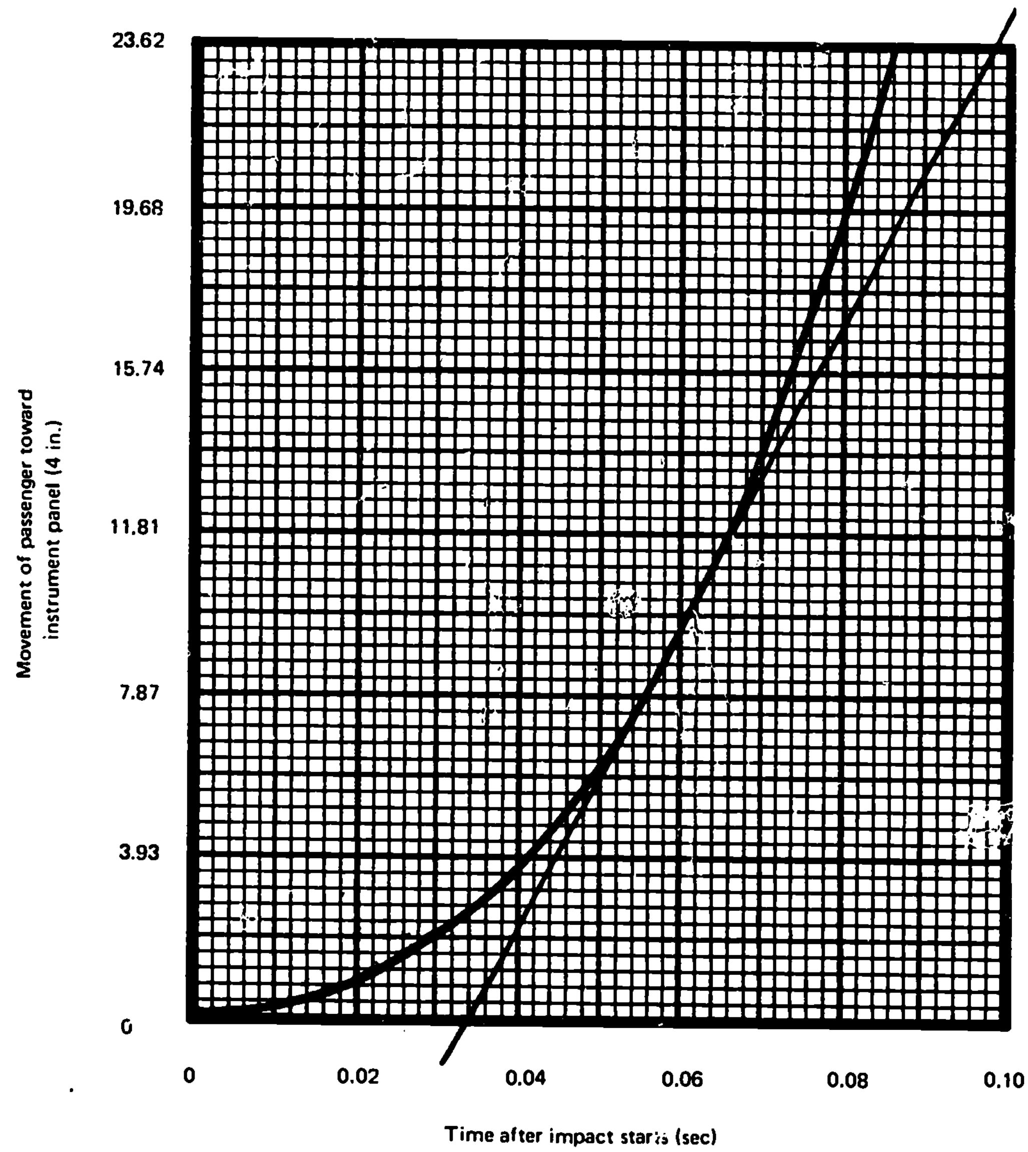
Answer

$v = [\Delta x/\Delta t]$ 0.06 sec = 37.5 ft/sec

Comments:

Note how fast the man is moving relative to the car which is in fact the object with which the man collides.

6. The driver of car 1 in the accompanying picture is moving with a speed of 30 mph in the direction indicated. He has a radar set-up in his car which tells him that car 2 is approaching him with a relative velocity of 50 mph. How fast is car 2 actually moving relative to the highway?



After R. L. Abbott

The New Zealand Medical Journal *66*, 1967, p. 431

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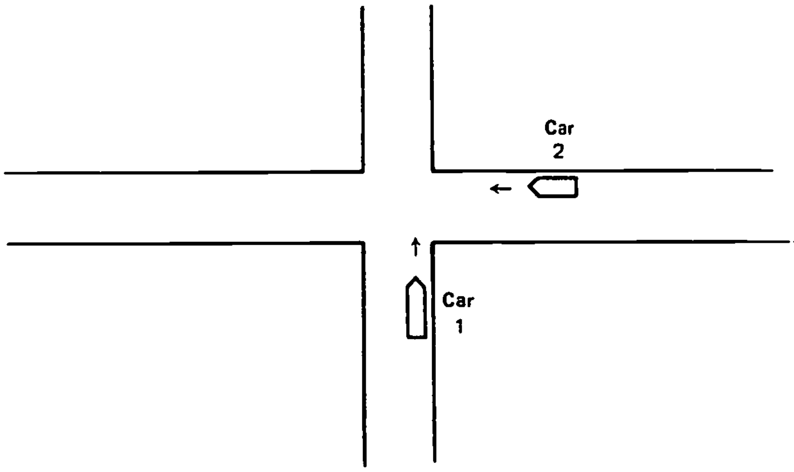

Answer

$V_2 = +40$ mph.

Comments:

The speed with which two cars approach one another at right angles is greater than either speed but less than the sum of the two.

7. We would like to determine the relative velocity between two cars whose speeds are: car 1 → 30 mph; car 2 → 60 mph. Consider the following situations:

   a) 1 and 2 are going the same direction.

   b) 1 and 2 are going in opposite directions.

   c) 2 approaches 1 at an angle of 45° from head-on.

Answer

a) $V_R = 30$ mph

b) $V_R = 90$ mph

c) $V_x = 30(1 + \sqrt{2})$ mph $= 72$ mph
$V_y = +(30\sqrt{2})$ mph $= +42$ mph

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Velocity and Acceleration

LABORATORY 1

I. PURPOSE

A. To demonstrate velocity and acceleration and how these quantities may be measured.

B. To give the student an appreciation of how large some familiar velocities and acceleration of automobiles are.

II. EQUIPMENT

One air track (including carts and sparking equipment to measure velocities)

Some graph paper

An automobile

Several stopwatches

III. PROCEDURE

A. Using the airtrack determine different cart velocities using the spark timer. Convert the velocities to mph. Also try to estimate some of those velocities on purely an intuitive basis, and record your guesses for later comparisons with the actual velocities.

B. By tilting the track at various angles, determine some decelerations.

C. Compare the decelerations due to the collisions of carts bumping end to end with those in part B.

D. Using an automobile, some markers and stopwatches, determine several car velocities.

E. Station several students with stopwatches at equal intervals along a street where the car may be accelerated uniformly.

Determine the acceleration of the car from the times recorded by the students and ask each student to guess some intermediate velocities (to be compared with those measured by a clock carrying passenger in the car).

IV. QUESTIONS

A. What two basic quantities are involved in order to describe motion?

B. What factors limit the working velocities of the air track carts?

C. Extend the results of parts III.B and E to accelerations experienced in automobiles during everyday driving and during a collision situation.

D. If you were a policeman and you were trying to maximize the number of tickets given for speeding, considering part III.A, where would you station yourself?

V. COMMENTS

A. People in general do not realize how fast objects are moving. At 10 or 20 mph, there is still danger involved in an automobile accident since a localized blow may cause severe damage to the body even if it is not too large in total force.

B. The decelerations experienced in collisions are much larger than those occurring in everyday driving. To minimize decelerations experienced in a collision, one should prevent impacts such as head-windshield or chest-dashboard (which occur very quickly) by wearing a seat belt which permits a less dangerous distribution of the forces present as also minimizes these forces by decelerating the passenger with the car.

C. When estimating such things as velocities, one should remember that they can seem quite relative and immediate prior experience may be quite important for such estimates.

REVIEW

The review sections in this book may be reproduced and distributed to students for use as a unit review.

The material is designed as programmed instruction. The students should be instructed to solve each problem before checking the answer on the next page, in order to obtain maximum benefit from the material.

VELOCITY

Review

Concept Review

1. **Observer** – An observer is a person who is capable of making any physical measurement and who is represented by a coordinate system.

2. **Time** – The elapsed time according to an observers clock will be represented by the variable "t". Changes in time, or time intervals, will be represented by Δt or by T.

3. **Position** – The position of an object which is moving along a straight line (as measured by a certain observer) is the distance of the object from the origin of the observer, multiplied by a plus or minus 1. We will agree that all positions on one side of the observer are positive ("rightside") and on the otherside ("leftside") are negative. We will represent the position of an object along a straight line by the variable x. Since this position will generally depend on time, we will write $x(t)$ meaning that the object is at the position x at the time t.

4. **Change in position** – The change in the position of an object moving along a straight line between the times t and $t + \Delta t$ is $\Delta x = x(t + \Delta t) - x(t)$.

5. **Average velocity** –
$$V_{AVE} = \Delta x / \Delta t = [x(t + \Delta t) - x(t)]\ [(t + \Delta t) - t] = [x(t + \Delta t) - x(t)] / \Delta t$$

6. **Instantaneous velocity** – The velocity that an object has at any time.
$$V(t) = \lim_{\Delta t \to 0} \left\{ [x(t + \Delta t) - x(t)] / \Delta t \right\}$$

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Problem Review

1. A car travels along an east-west road. An observer who is located on the road makes the following position and time measurements for the car:

Table 1

x (ft)	t (sec)
-100	0
-96	1
-84	2
-64	3
-36	4
0	5
44	6
96	7
156	8
224	9
300	10

x = Position of car relative to observer

t = Time of position measurement

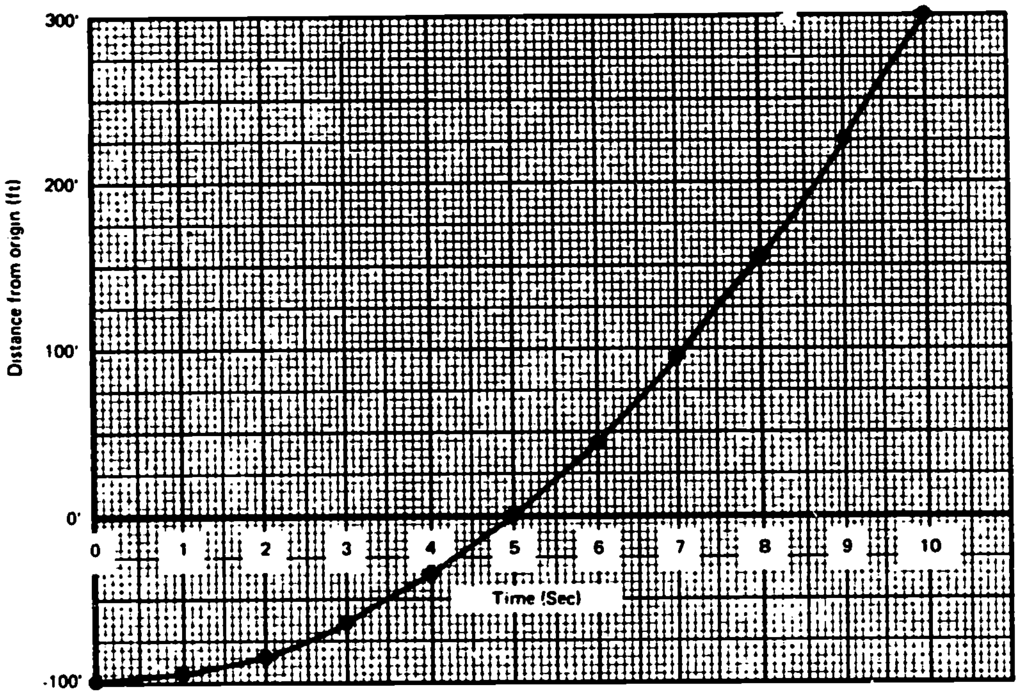
a) What is the meaning of the negative values of position?

b) Graph the points listed in table 1 and devise a simple way of estimating the position of the car at *any* time between 0 sec and 10 sec.

Answer 1

a) Negative values of position mean that the car is on the side of the observer chosen as negative. (Negative real axis)

b)



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2. Referring to Table 1:

   a) Determine the change in position between each time listed. (Put this in table form.)

   b) Determine the average velocity of the car between each of the times given.

   c) What is the relationship between these calculated values of average velocity and the slope of the straight lines given in answer 1(b)?

Answer 2

a) Δx = change in position

Δx (ft)	Δt (sec)
4	$0 \rightarrow 1$
12	$1 \rightarrow 2$
20	$2 \rightarrow 3$
28	$3 \rightarrow 4$
36	$4 \rightarrow 5$
44	$5 \rightarrow 6$
52	$6 \rightarrow 7$
60	$7 \rightarrow 8$
68	$8 \rightarrow 9$
76	$9 \rightarrow 10$

b) $V_{AVE} = \Delta x / \Delta t$

Since $\Delta t = 1$ sec Then

V_{AVE} (ft/sec)	Δt (sec)
4	$0 \rightarrow 1$
12	$1 \rightarrow 2$
20	$2 \rightarrow 3$
28	$3 \rightarrow 4$
36	$4 \rightarrow 5$
44	$5 \rightarrow 6$
52	$6 \rightarrow 7$
60	$7 \rightarrow 8$
68	$8 \rightarrow 9$
76	$9 \rightarrow 10$

c) The slopes of the straight lines given in solution 1(b) are just the values of $\Delta x / \Delta t$ listed in solution Table 2.

3. A room is 30 ft. long. A toy car traveling at a *constant* velocity of 20 mph goes across the room. How long does it take the car to cross?

Answer 3

Using the approximation that 3/2 ft/sec = 1 mph we find that 20 mph = 30 ft/sec, so it takes the car approximately 1 sec to cross the room.

4. A car collides with a telephone pole. Because the driver is wearing a seat belt he stops with the car after moving a distance of 3 ft. during the collision. If the collision takes .2 seconds,

   a) What is the average velocity of the car during the collision. Give answer in ft/sec and miles per hour.

   b) How do these velocities compare to those of a man who runs the hundred yard dash in 10 sec?

Answer 4

a) Since the passenger is constrained to move with the car the "center" of the car also moves 3 ft in .2 sec. Hence, $V_{AVE} = \Delta x/\Delta t = 3\text{ ft}/.2\text{ sec} = 15\text{ ft/sec} = 10\text{ mph}$

b) 100 yards = 300 ft

$V_{AVE} = 300\text{ ft}/10\text{ sec} = 30\text{ ft/sec} = 20\text{ mph}$

SOMETHING TO THINK ABOUT

If the passenger were not seat belted, how would the collision time and collision distance (the time and distance over which the body is being acted upon by stopping forces) change?

5. We would like to measure the instantaneous velocity of a car, (the actual velocity of the car at a certain time). The car moves along a straight road and at $t = 5$ sec, according to our stationary observer, the front bumper of the car passes a white line marked on the highway. The line is 125 ft from the observer. The following table gives further times and positions of the bumper at those times.

x (ft)	t (sec)
125.0	5.0
125.005	5.0001
125.025	5.0005
125.05	5.001
125.25	5.005
125.50	5.01
130.05	5.1
180.00	6.0

a) Determine approximately the instantaneous velocity of the car at $t = 5$ sec.

b) Is the car moving at a constant velocity?

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Answer 5

a) The instantaneous velocity at any time t is defined as:

$$V(t) = \lim_{\Delta t \to 0} [x(t + \Delta t) - x(t)]/\Delta t$$

The limit symbol means that we must look at the value approached by $[x(t + \Delta t) - x(t)]/\Delta t$ as Δt becomes arbitrarily small or approaches zero.

Let t = 5 sec and let us form the following table for $\Delta x = x(5 + \Delta t) - x(5)$ and Δt:

Δx (ft)	Δt (sec)
.005	.0001
.025	.0005
.050	.001
.250	.005
.500	.01
5.05	.1
55.00	1.0

We can approximate the limit idea by looking at values of $V_{AVE} = \Delta x/\Delta t$ for successively smaller and smaller values of Δt.

$\Delta x/\Delta t$ (ft/sec)	Δt (sec)
55.5	1.0
50.56	.1
50.0	.01
50.0	.005
50.0	.001
50.0	.0005
50.0	.0001

We estimate the value of the car's velocity at t = 5 sec to be 50 ft/sec.

b) The velocity is not a constant because of the values of 50.56 ft/sec and 55.5 ft/sec in our table.

6. Along a drag strip a car is found to have its position x related to its time of measurement t in the following fashion: $x(t) = 5t^2$. Determine the instantaneous velocity of the car at $t = 5$ sec.

Answer 6

$$
\begin{aligned}
v(t) &= \lim_{\Delta t \to 0} \left\{ [x(t+\Delta t) - x(t)]/\Delta t \right\} \\
&= \lim_{\Delta t \to 0} \left\{ [5(t+\Delta t)^2 - 5t]/\Delta t \right\} \\
&= \lim_{\Delta t \to 0} \left\{ [5t^2 + 10t(\Delta t) + 5(\Delta t) - 5t]/\Delta t \right\} \\
&= \lim_{\Delta t \to 0} [10t + 5(\Delta t)]
\end{aligned}
$$

As $\Delta t \to 0$ the term $5(\Delta t)$ goes to zero.

So that $v(t) = 10t$ (ft/sec)

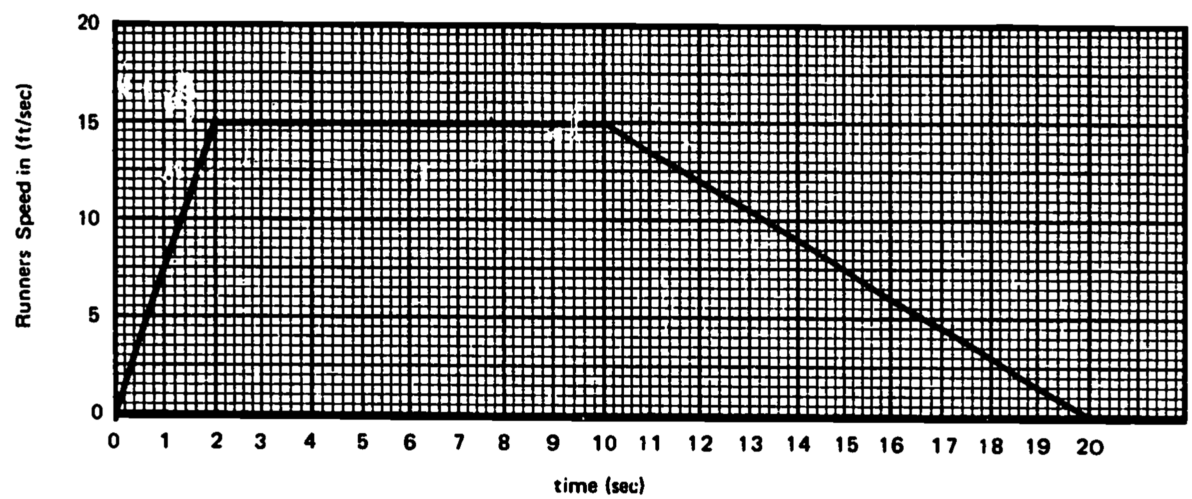
at 5 sec, $v(5) = 50$ ft/sec

SOMETHING TO THINK ABOUT

If the driver of a car in a drag race *always* drives his car at a constant velocity (except at the very beginning and at the very end), would seat belts be necessary for him?

For the car described in this problem do you think the driver needs to wear a seat belt?

7. A man runs along a road. If we graph his velocity against time we find that it looks as follows:



a) At what times was the man not moving?

b) When was the man running fastest?

c) What is the change in the man's velocity between $t = 0$ and $t = 10$ sec? Between $t = 10$ sec and $t = 20$ sec?

d) How far did the man run and how long did it take?

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Answer 7

a) The man was not moving for t less than 0 sec and t greater than 20 sec.

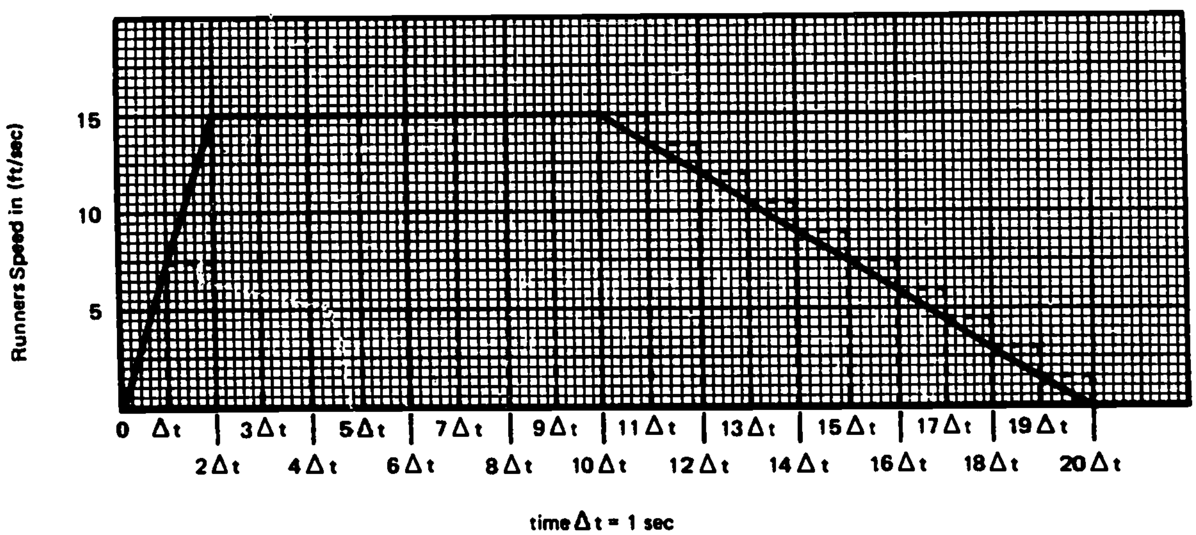
b) The man runs fastest between 2 sec and 10 sec.

c) $\Delta V = V(10) - V(0) = 10 \text{ mph} - 0 \text{ mph} = 10 \text{ mph} \approx 15$ ft/sec

$\Delta V = V(20) - V(10) = 0 - 10 \text{ mph} = -10 \text{ mph} \approx -15$ ft/sec

d) To determine how far the man moved we note the following: The change in the man's position between t and $t + \Delta t$ (when Δt is a *very* small time interval) is to a very good approximation $\Delta x \approx V(t)\,\Delta t$. Suppose now we do the following. Divide the time interval of $0 \rightarrow 20$ sec into a large number of smaller equal parts (say, for example, into 20 parts). Then the length of each time interval is $\Delta t = 1$ sec. Calculate Δx_1 (the first change in position) using $V(0)\,\Delta t$. Then calculate $\Delta x_2 = V(0 + \Delta t)\,\Delta t$ and $\Delta x_2 = V(0 + 2\,\Delta t)\,\Delta t$ and so on for all times between 0 and $20\,\Delta t = 20$ sec. Then total change in position is just $\Delta x_1 + \Delta x_2 + \ldots \Delta X_{20}$.

To see what this means with a picture lets look at the graphs of V(t) *vs.* t. We have marked off the 19 times Δt, $2\,\Delta t$, $3\,\Delta t$, . . . etc.



The quantity $V(t)\,\Delta t$ is actually the area of a rectangle plotted on the V(t) *vs* t graph. The base of the rectangle extends between t and $t + \Delta t$ and the height is V(t). As you can see the sum $\Delta x_1 + \Delta x_2 + \ldots \Delta x_{20}$ forms an approximation to the area between the V(t) curve and the horizontal axis.

Suppose we divide our time interval of 20 sec into not 20 parts but 20×10^6 parts and once again add up the areas $V(t)\,\Delta t$ for all times 0, Δt, $2\,\Delta t$ . . . $20 \times 10^6\,\Delta t$. We obtain a much better approximation to the area under the V(t) curve and our distance estimate becomes much more precise. The best result then appears to come if we divide the 20 sec interval into a very large number of parts. The change in position of the man in this limit becomes exactly the area under the V(t) curve.

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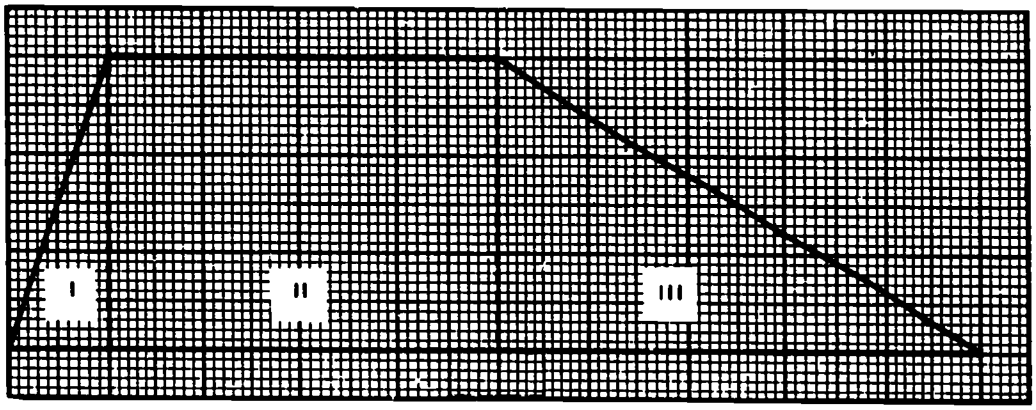
Consider our V(t) curve again:



To get the total areas we say $A_{TOTAL} = A_I + A_{II} + A_{III}$

$A_I = \frac{1}{2}(2 \text{ sec}) \times (15 \text{ ft/sec}) = 15 \text{ ft.}$

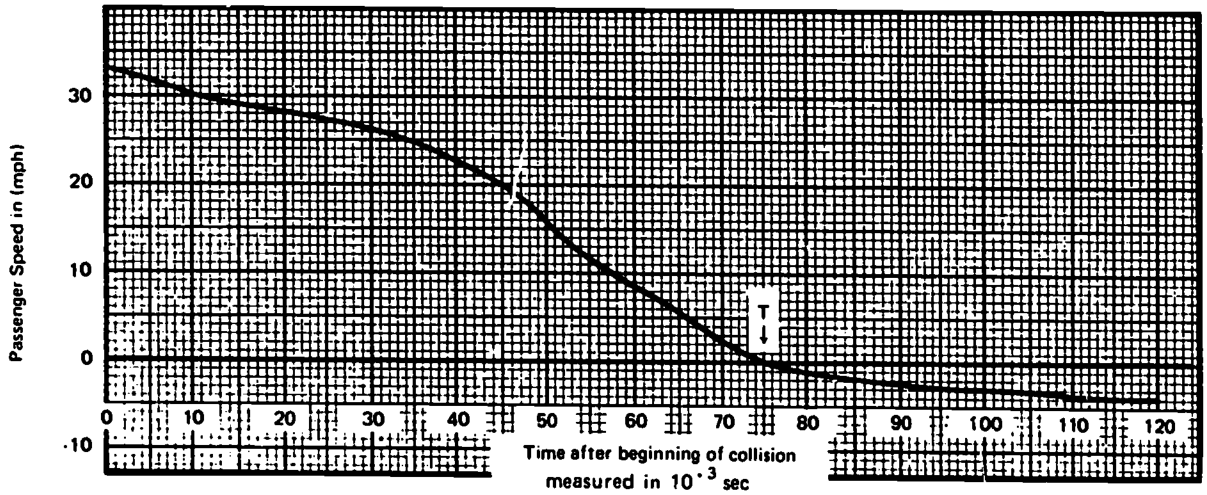
$A_{II} = 8 \text{ sec} \times 15 \text{ ft/sec} = 120 \text{ ft.}$

$A_{III} = \frac{1}{2}(10 \text{ sec}) \times (15 \text{ ft/sec}) = 75 \text{ ft.}$

$A_{TOTAL} =$ 210 ft.

The time of travel was 20 sec.

8. In a head-on collision of a car with a barrier, the velocity of a seat belted passenger (according to a stationary observer) actually looks as follows:



a) What is the change in the velocity of the passenger between t = 0 and t = T (sec)? Compare this answer with those of 7(c).

b) Determine the distance the passenger travels during the collision, (from t = 0 sec to t = .120 sec).

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Answer 8

a) $\Delta V = V(T) - V(0) = 0 - v(0) = -33 \text{ mph} = -49.5 \text{ ft/sec}$

b) To find the distance travelled by the passenger during the collision, we need to determine the area under the above V(t) curve. Note that if V(t) is negative (the tail on the above curve) the area units calculated $V(t)\Delta t$ are also negative. The total area under the above curve is roughly 2.07 ft, (remember to convert the passengers' velocity from mph to ft/sec).

SOMETHING TO THINK ABOUT

If the passenger were not wearing a seat belt what changes would you expect in the V(t) curve? Explain these changes in terms of what is happening to the passenger inside the car.

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ACCELERATION

Demonstrations

Demonstration 1

Use rubber bands to stop a toy car traveling on the floor and try to determine the stopping time for various strength rubber bands. Calculate the average accelerations.

Demonstration 2

Mark off equal distances on a wall along which a ball can be dropped. Determine the average velocities for the various intervals and then determine the acceleration.

Demonstration 3

Try to estimate stopping times for "collisions", the type of which one sees every day (man hitting a baseball, a boy hitting the ground after jumping, a headache ball hitting a building, etc.)

ACCELERATION

Examples and Problems

1.E During a head on collision at 30 mph a car is found to travel a distance of 2 ft because of the crushing of the car before it comes to rest:

a) Determine the average acceleration of the car during the collision in ft/sec^2. In g's.

b) Assuming the acceleration is a constant over the time interval of stopping, determine the time for the car to stop.

Answer

a) $2ax = V_F^2 - V_o^2$
$x = 2$ ft; $V_F = 0$ mph; $V_o = 30$ mph $= 44$ ft/sec
$\therefore a = 484$ ft/sec^2 $= 15$ g's

b) $T = V_o/a = 0.091$ sec

Comments:

Note the average deceleration is 15 g's. The instantaneous deceleration of the passengers (during the collision) is often much more. Also note the contact time during the collision is about the reaction time of an average person (0.1 sec).

2.E In a head-on collision at 33 mph the speed of an unrestrained passenger in the front seat (relative to the seat) looks roughly as shown in the graph on the next page.

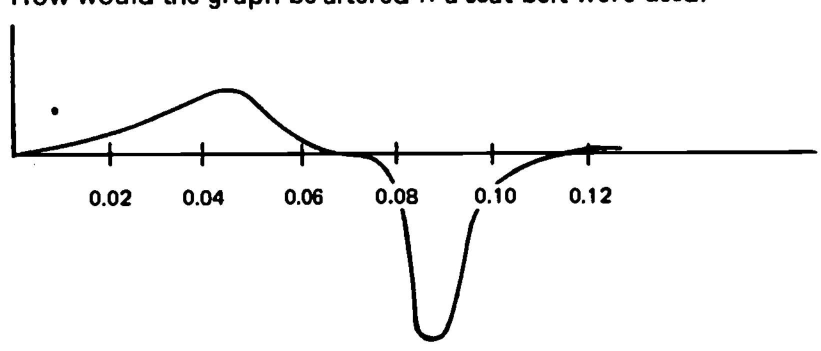
a) Graph the acceleration (relative to the car) as a function of time.

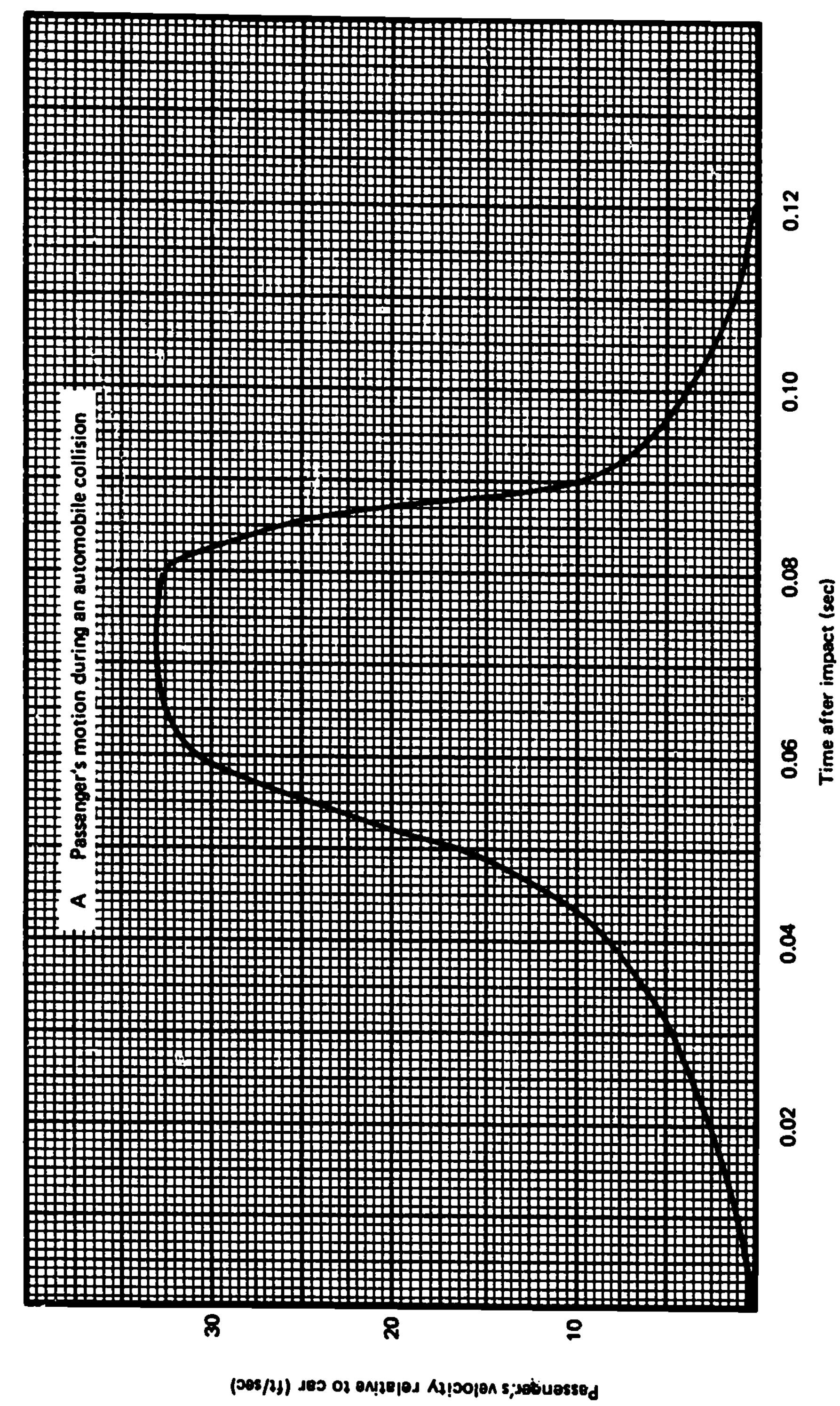
b) Explain the graph in terms of what is happening to the passenger.

c) How would the graph be altered if a seat belt were used?

Answer

a)




A Passenger's motion during an automobile collision
0.02
0.04
0.06
0.08
0.10
0.12
Time after impact (sec)
30
20
10
Passenger's velocity relative to car (ft/sec)

36

b) The car has a high rate of deceleration from 0 to 0.06 sec and the man is moving faster and faster relative to the car. At 0.08 sec the man hits the windshield and/or dashboard and decelerates very rapidly until at about 0.090 sec he is coming to rest relative to the car (and the road also since the car will be almost stopped at this time).

c) The man's deceleration (relative to the car) would not be peaked like in "a)" since he is bound to the car with the seat belt and thus his deceleration relative to the car would be very small. However, both the car and the man will experience large decelerations relative to a stationary observer located outside the car.

3. A 20 lb. box sits in the back seat of a convertible. The car, travelling at 20 mph hits a pothole in the road and the box is thrown out.

    a) How far from its initial position at the time of the impact is the box when it hits the ground? Assume that the seat of the car is 3 ft above the pavement and that the box is ejected at an angle of 30° with the horizontal.

    b) How fast is the box moving when it hits?

Answer

a) $V_{yo} = (30 \text{ ft/sec}) \sin 30° = 15 \text{ ft/sec}; a = -32 \text{ ft/sec}^2$

$y = V_{yo}t + \frac{1}{2} at^2,$

$\therefore T = 1.1 \text{ sec}$

$x = V_xT, V_x = (30 \text{ ft/sec}) \cos 30° = 26 \text{ ft/sec}$

$\therefore x = 29 \text{ ft.}$

b) $2ay = V_y{}^2{}_F - V_y{}^2{}_o$

$a = -32 \text{ ft/sec}^2, y = -3 \text{ ft}, V_{yo} = 10 \text{ mph} = 15 \text{ ft/sec}$

$V_y = 20 \text{ ft/sec}$

$\therefore V = 33 \text{ ft/sec}$

Comments:

Note that the answers to (a) and (b) do not depend on the mass of the box. Hence the box could have been a back seat passenger. The speed of 33 ft/sec is slightly greater than the average speed need for a 10 sec – 100 yard dash.

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4. A car accelerates from 0 to 60 mph in 10 sec.

a) What is its average acceleration?

b) Suppose that the car while traveling at 60 mph hits a wall and stops in 0.1 sec. What is the average deceleration?

c) Convert the answers in a) and b) to "g" units.

Answer

a) $a = 8.8\ \text{ft/sec}^2$

b) $a = 880\ \text{ft/sec}^2$

c) $8.8\ \text{ft/sec}^2 = .27\ g$
$880\ \text{ft/sec}^2 = 27\ g$

5. During a fire a man is forced to jump from the 4th floor of a building (about 30 ft from the ground).

a) What is the man's acceleration during the fall in ft/sec^2 and in "g" units?

b) What is his speed just before he contacts the ground, (neglect air friction)? Express your answer in mph and fps.

c) If the man stops in .05 sec when he hits the ground, what is his average deceleration in ft/sec and in g units?

Answer

a) $1\ g = 32\ \text{ft/sec}^2$

b) $v_F{}^2 = v_o{}^2 + 2\ ax$
$v_o = 0 \quad x = 30\ \text{ft} \quad a = 32\ \text{ft/sec}^2$
$\therefore v = 43.8\ \text{ft/sec} = 30\ \text{mph}$

c) $a = v/T = 876\ \text{ft/sec}^2 = 27\ g$

Comments:

Compare the deceleration here with that experienced by the passenger in problem 2.

6. If a car decelerates uniformly from 40 mph to 0 mph in 80 msec. (0.080 sec):

   a) What is the deceleration?

   b) What is the distance traveled during this period?

Answer

a) $T = .08$ sec
   $a = (v_F - v_o)T = 23$ g

b) $d = v_oT + \frac{1}{2} aT^2 = 2.4$ ft

Comments:

These results are quite similar to actual collision parameters. That is: for a head-on collision with a barrier at 40 mph it takes a time of about 0.1 sec with a crushing distance of about 2 ft for the car. The average deceleration is great and the peak acceleration during the collision is two or three times as great.

7. A batter hits a pitched ball for a home run. The ball is hit when it is moving horizontally toward the plate at 100 ft/sec. The ball leaves the bat at approximately 120 ft/sec at an angle of 30° with the horizontal. The impact time is 0.1 sec. Determine:

   a) The average acceleration of the ball during impact.

   b) Express the magnitude of the acceleration in part (a) in "g" units.

Answer

a) $a_x = \dfrac{\Delta v_x}{\Delta t}$ $\qquad$ $a_y = \dfrac{\Delta v_y}{\Delta t}$

   $a_x = -2040$ ft/sec^2 $\qquad$ $a_y = 600$ ft/sec^2

b) $a = \sqrt{a_x{}^2 + a_y{}^2}$
   $= 67$ g

Comments:

Note the tremendous acceleration given to the ball which weighs 5 ounces when it is hit by a bat which is much heavier than it is. The acceleration involved here is about the same as the peak deceleration in a head-on barrier collision for a car traveling at 60 mph.

8. During a head-on barrier collision the deceleration of a car as a function of time was found to be as shown on the accompanying graph (following page).

   a) What is the speed of the car before the collision?

   b) Sketch the speed of the car vs. time.

Answer

a) V_o = 49 ft/sec = 33 mph (This answer is obtained by finding the area under the deceleration curve)

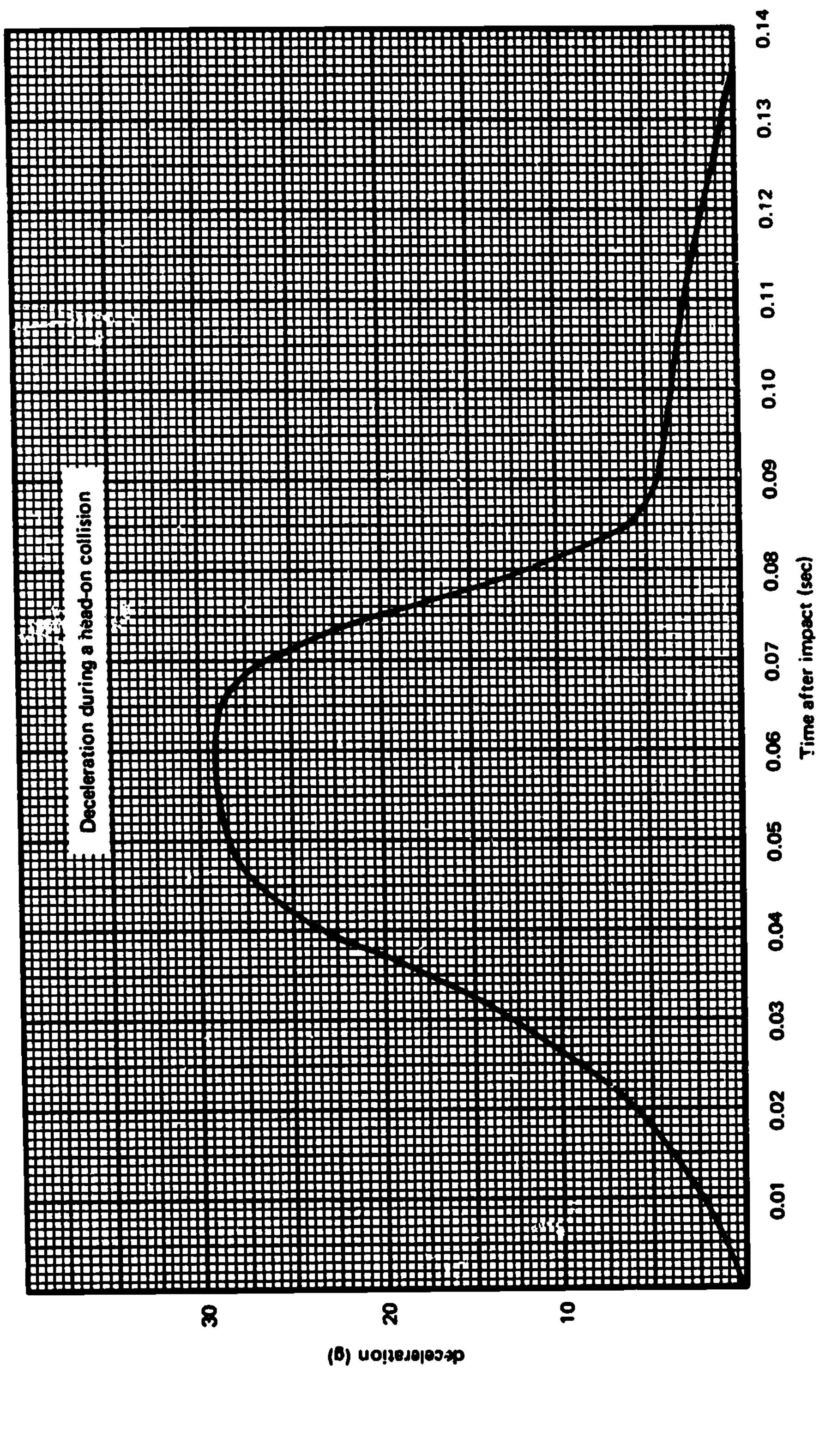
b) $V(t) = V_o -$ area (t) [Here Area (t) means the area under the deceleration curve between t = 0 and t]

T	V
.01	49 ft/sec
.02	48 ft/sec
.03	47 ft/sec
.04	44 ft/sec
.05	38 ft/sec
.06	30 ft/sec
.07	23 ft/sec
.08	10 ft/sec

T	V
.09	5.1
.10	3.2
.11	2.0
.12	1.0
.13	0.3
.14	0
.15	0

Comments:

Note that although the average deceleration is about 480 ft/sec^2 or 15 g's, the peak deceleration is about 30 g. That the peak deceleration should exceed the average deceleration by a factor of 2 or 3 is a common result for head-on barrier car collisions.


Deceleration during a head-on collision
deceleration (g)
30
20
10
Time after impact (sec)
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14

REVIEW

The review sections in this book may be reproduced and distributed to students for use as a unit review.

The material is designed as programmed instruction. The students should be instructed to solve each problem before checking the answer on the next page, in order to obtain maximum benefit from the material.

ACCELERATION

Review

Concept Review

1. **Velocity** — The rate at which the position of an object is changing with time. The velocity of an object is often a function of time.

2. **Change in velocity** — If the velocity of an object is measured between two times t and $t + \Delta t$, then the change in the velocity ΔV between t and $t + \Delta t$ is $\Delta V = V(t + \Delta t) - V(t)$.

3. **Average acceleration** — $a_{AVE} = \Delta V / \Delta t = [V(t + \Delta t) - V(t)] / \Delta t$

4. **Instantaneous acceleration** — $a(t) = \lim_{\Delta t \to o} [V(t + \Delta t) - V(t)] / \Delta t$

Constant Acceleration Formulas for a Moving Body

1. $V(t) = V_o + a\,t$ — where $V(t)$ is velocity at any time t
   V_o is velocity at time $t = 0$
   a is the constant acceleration
   t is time

2. $x(t) = x_o + V_o t + ½ at$ — $x(t)$ is the position of the body as a function of time
   x_o is the body's position at $t = 0$

3. $V^2 - V_o^2 = 2a(\Delta x)$ — V = velocity at some time t
   V_o = velocity at $t = 0$
   a = constant acceleration
   Δx = change in body's positions between time $t = 0$ and t.

Note ________ The word uniform is often used in the following problems. If acceleration is uniform, then it is constant.

Problem Review

1. A car moving along a long straight road has a velocity of 10 ft/sec at $t = 5$ sec and 50 ft/sec at $t = 9$ sec.

   a) What is the change in the velocity during this time interval?

   b) What is the average acceleration of the car in units of ft/sec^2? In "g" units?

   c) What is the relationship between the average acceleration and a uniform acceleration operating over the same time interval?

Answer 1

a) $\Delta V = V(t + \Delta t) - V(t)$

$t = 5$ sec

$\Delta t = 4$ sec

$\Delta V = 50$ ft/sec $-$ 10 ft/sec = 40 ft/sec

b) $a_{AVE} = \Delta V/\Delta t = 40$ ft/sec/4 sec = 10 ft/sec^2

In "g" units a_{AVE} = 10 ft/sec^2/32 ft/sec^2 = 1/3 g

c) If we change the initial time $t = 5$ sec to $t = 0$ and the final time $t = 9$ sec to $t = 4$ sec (subtract 5 sec from each time) then formula (1) for constant acceleration gives

$V(4) = V_o + 4a$ or

$a = V(4) - V_o/4$ which is just our expression for the average acceleration.

2. A car stops suddenly in 1/10 sec from a speed of 30 mph. Two people are riding in the car. The driver wears a seat belt, the other person does not. The driver since he wears a seat belt stops with the car. Assuming constant deceleration:

   a) Find the average acceleration undergone by the driver in ft/sec^2? in "g" units?

   b) Assume the acceleration is a constant equal to the average, how far does the driver move with respect to an outside observer. (Obtain this result using formulas (2) and (3).)

   The passenger does not wear a seat belt and does not stop with the car. Instead he stops all of a sudden when his body hits the dashboard. The stopping time is not .1 sec, but is roughly .02 sec.

   c) Determine the average acceleration of the passenger.

   d) If the passenger were brought to rest with uniform acceleration equal to this average acceleration, how far would he have moved?

*Note*________In this latter case (unrestrained passenger), the stopping distance results in a crushing of the body.

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Answer 2

a) $a_{AVE} = [V(t + \Delta t) - V(t)] / \Delta t = -45$ ft/sec/1/10 sec $= -450$ ft/sec^2

$a_{AVE} = -14\,g$

The minus sign indicates that the car is slowing down (decelerating).

b) From formula (2) we want to calculate

$\Delta x = x - x_o = V_x T + \frac{1}{2} a T^2$ when T = time to stop.

T = 1/10 sec

V = 30 mph = 45 ft/sec

$\Delta x = 45/10 - \frac{1}{2}(450) \times (1/100) = 4.5$ ft $- 2.25$ ft $= 2.25$ ft

From formula (3) we have

$V^2 - V_o^2 = 2a \Delta x$

a = 450 ft/sec^2

V_o = 45 ft/sec

V = 0 (stopping condition)

$-(45'/\text{sec})^2 / [-2 \times 450 \text{ (ft/sec)}] = 2.5 \text{ ft} = \Delta x$

*Note*_________This distance is the amount the car is crushed in the collision.

c) $a_{AVE} = [V(t + \Delta t) - V(t)] / \Delta t$

Let $V(t + \Delta t) = 0$ (stopping condition)

$V(t) = 45'/\text{sec}$ (unseatbelted passenger moves with the initial velocity of the car until he is forced to slow down)

$\Delta t = .02$ sec

$a_{AVE} = -2250$ ft/sec^2 $= -70\,g$

d) $\Delta x = V^2 - V_o^2/2a = -(45 \text{ ft/sec})^2 / (-2 \times 2250 \text{ ft/sec}^2) =$

= 5.4 inches

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3. To calculate the instantaneous acceleration of a car at a certain time of observation, say $t = 5$ sec, we determine the velocity both at $t = 5$ sec and $t = 5$ (sec) $+ \Delta t$ where Δt is now a small time interval.

Below are listed the velocity at various times. Can you predict how fast the velocity is changing with time at $t = 5$ (sec)?

V(t) (ft/sec)	t(sec)
50	5
50.0008	5.0001
50.008	5.001
50.08	5.01
50.8	5.1

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Answer 3

(Refer to problem (5) under velocity)

$$a(t) = \lim_{\Delta t \to 0} [V(t+\Delta t) - V(t)]/\Delta t$$

We will approximate limit by looking at $\lim_{\Delta t \to 0} [V(t+\Delta t) - V(t)]/\Delta t$

for smaller and small time intervals Δt.

$[V(t+\Delta t) - V(t)]/\Delta t$	t
8.0	.0001
8.0	.001
8.0	.01
8.0	.1

For this problem it is not really necessary to let $\Delta t \to 0$ since $a(t) = 8$ ft/sec^2 — a constant.

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4. A car starts from rest and accelerates uniformly from zero to 60 mph in 10 sec.

   a) What is the acceleration in ft/sec^2? in "g" units?

   b) How far does the car travel in 10 sec?

   Suppose the car is forced to stop on wet pavement in a normal distance of 38 car lengths or 607 ft. If the car decelerates uniformly:

   c) What is the deceleration in ft/sec^2? in "g" units?

Answer 4

a) Using formula (1) we obtain $V = aT$

$V = 60$ mph $= 90$ ft/sec $= aT$

If $V = 60$ mph $= 90$ ft/sec and $T = 10$ sec

Then $\Delta V/T = 9$ ft/sec^2 $= 9/32\,g = .28\,g$

b) $V^2 - V_o^2 = 2a\Delta x$

$V_o = 0$

$V = 90$ ft/sec

$a = 9$ ft/sec^2

$\Delta x = (90 \text{ ft/sec})^2/(18 \text{ ft/sec}^2) = 450$ ft.

c) $V^2 - V_o^2 / 2\Delta x = a = -d$ (d is the deceleration)

$d = (90 \text{ ft/sec})^2/[2(607 \text{ ft})] = 6.7 \text{ ft/sec}^2 = .21$ g.

SOMETHING TO THINK ABOUT

Note that the accelerations experienced are a little larger than those commonly experienced in a car yet are still just fractions of a "g". In accidents, the body can experience accelerations that are often 20 g's and higher.

5. A boy jumps off a 10 ft. high brick wall.

   a) What is his acceleration in g's?

   b) What is the boy's speed just before hitting the ground in ft/sec and mph?

   c) If he stops in 1/10 sec after coming in contact with the ground, what is his average deceleration in ft/sec^2? in g's?

Answer 5

a) The acceleration in free fall (after the boy jumps and before he hits) is 1 g = 32 ft/sec^2

b) The boy's speed just before hitting is obtained from

$V^2 - V_o{}^2 = 2a \Delta x$

Let V = unknown

$V_o = 0$

$a = 32\ ft/sec^2$

$\Delta x = 10\ ft$

$V^2 = 2 \times 32 \times 10\ (ft/sec)^2$ units

$V = 8\sqrt{10}\ ft/sec = 25.3\ ft/sec = 16.9\ mph.$

c) Using formula (1) we obtain

$\Delta V/T = a \quad = - d$

$d = 25.3\ ft/sec/\ 1/10\ sec = 253\ ft/sec^2 = 8\ g.$

SOMETHING TO THINK ABOUT

The average deceleration of a restrained passenger in a head-on car collision at 17 mph with a barrier is the same as the answer in (c).

6. In a set of stopping experiments at 40 mph, various cars come to rest at different stopping distances. Assuming that a constant deceleration is employed:

   a) Can you predict the curve representing the relationship between deceleration and stopping distance for all cars in the experiment?

   b) If most collisions occur for stopping distances of 3 ft. and less, show that portion of the curve corresponding to these conditions.

   c) What accelerations are involved in part (b)?

Answer 6

a) $V^2 - V_o{}^2 = 2a\,x$

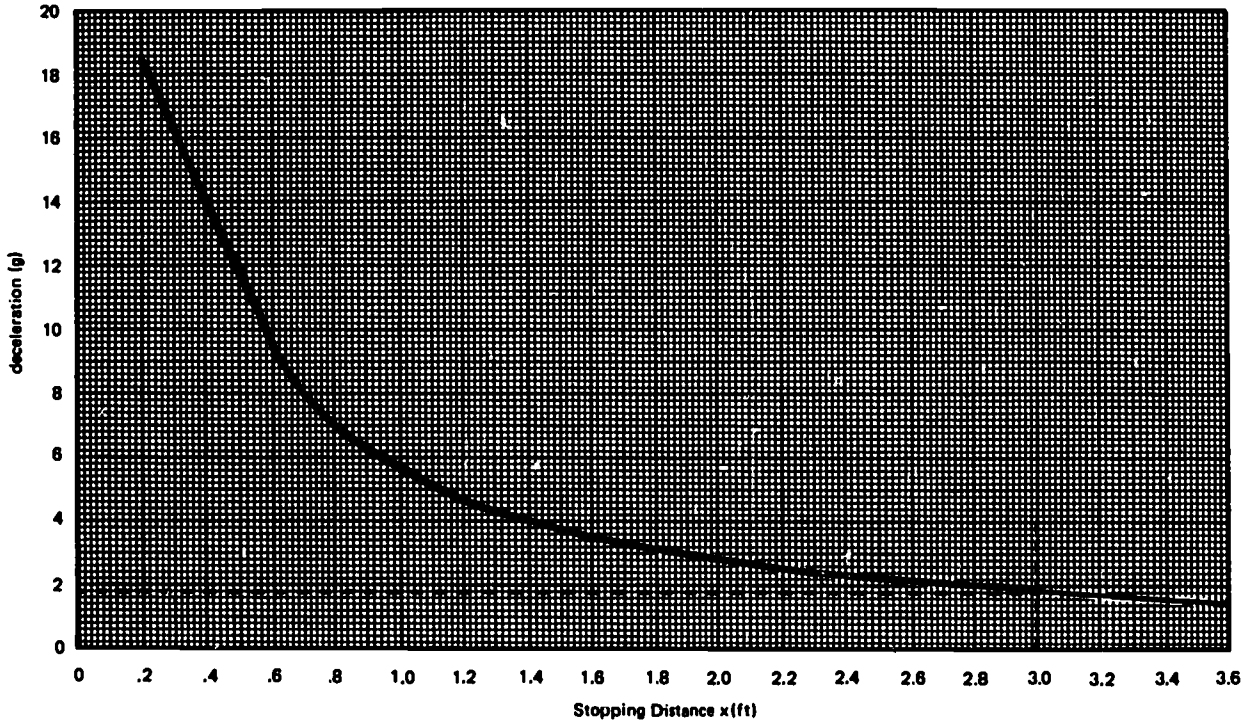
$V = 0$

$V_o = 40$ mph = 60 '/sec

a = acceleration = – d

x = stopping distance

d = V / 2x = 3600/2x = 1800/x ft/sec = (5.6 g – ft)/x where x is measured in feet.



b) See graph in part (a).

c) d greater than 1.87 g

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7. The velocity of an experimental sled is measured on a certain time period and is found to roughly obey the following equation: (This is actually a reasonably good approximation.)

$V(t) = 160 - 8000t^2 \quad 0 \leqslant t \leqslant .1$ sec

$V(t) = 80 - 8000(.1 - t)^2 \quad 0.1 \leqslant t \leqslant .2$

a) Determine the instantaneous deceleration as a function of time.

b) Graph this deceleration.

Answer 7

a) The instantaneous acceleration $a(t) = \underset{\Delta t \to 0}{\text{Limit}} \; [V(t + \Delta t) - V(t)]/\Delta t$

For $0 \leqslant t \leqslant .1$ $\quad a(t) = \underset{\Delta t \to 0_o}{\text{Limit}} \; (160 - 8000(t + \Delta t)^2) - (160 + 8000\,t^2)/\Delta t$

$= \underset{\Delta t \to 0}{\text{Limit}} \; [(160 - 8000t - 1600t \;\; t - 8000(\;\; t) \;\;) - (160 - 8000t \;\;)]/\Delta t$

$a(t) = \underset{\Delta t \to 0}{\text{Limit}} \; (-16000\,t - 8000(\Delta t) \;\;)$

$= -16000\,t$

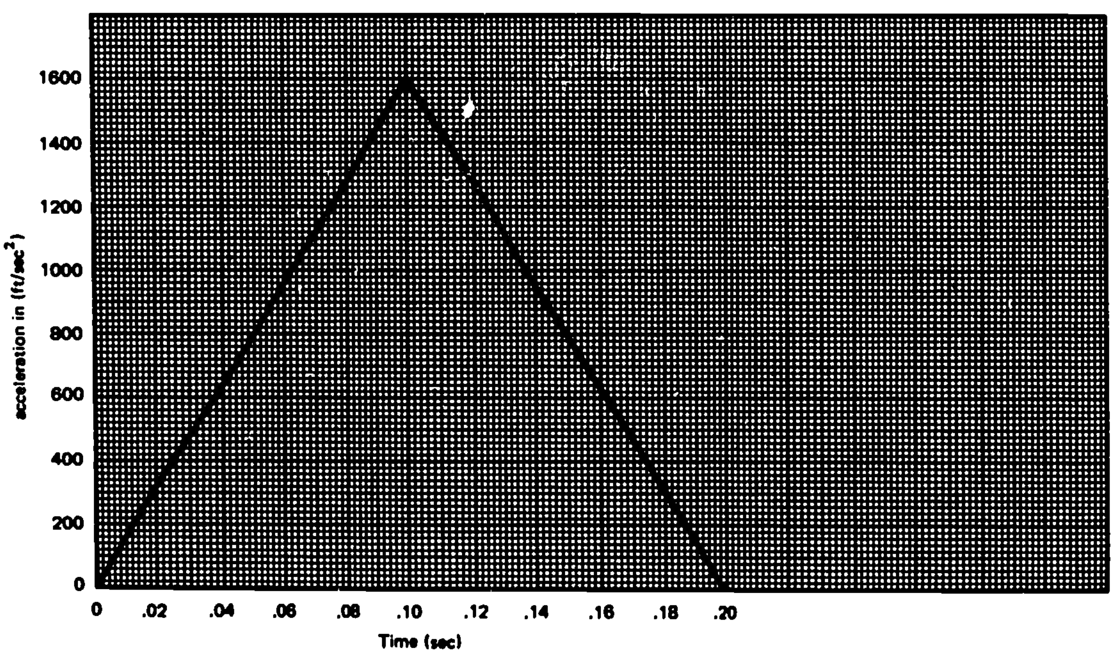
$d(t) = -\,a(t) = 16000\,t$

For $.1 \leqslant t \leqslant .2$

$a(t) = \underset{\Delta t \to 0}{\text{Limit}} \; (\; (80 - 8000(.1 - (t + \Delta t) \;\;)^2) - (80 - 8000(.1 - t)^2) \;\;)/\Delta t$

$a(t) = 1600 - 16000(t - .1)$

b)



SOMETHING TO THINK ABOUT

What is the ratio of peak deceleration to average deceleration for this curve?

MOMENTUM AND IMPULSE

Demonstrations

Demonstration 1

Using an air track and a rubberband stop, demonstrate that large masses have more momentum than smaller masses moving at the same speed. Repeat for equal masses and different velocities.

Demonstration 2

Model car-car and car-truck collisions on an air cart and attach passengers (blocks of wood) with a seat belt (rubber band). Can you tell which passengers have the most force acting on them during a collision?

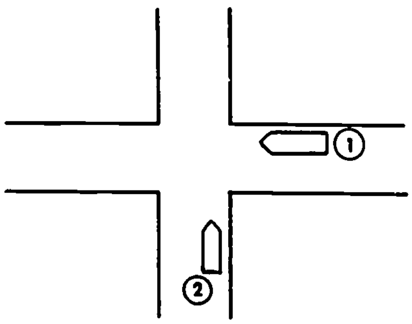
Demonstration 3

Tilt an airtrack upward and start a cart moving upward from the low end. Time how long it takes for the cart to stop and show that the area under the force *vs* time curve (assuming that only the gravity force acts) is just equal to the change in momentum of the cart.

MOMENTUM*

Examples and Problems

1.E Two cars collide at a 90° intersection. After the collision they remain locked together. Given the following data:



	weight	speed
Car 1	2000 lbs	30 mph
Car 2	1500 lbs	50 mph

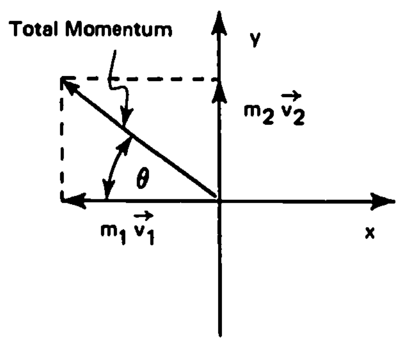
a) Construct a vector diagram to show the total momentum of the two car system before the collision.

b) In what direction is the final momentum of the system? Show graphically and compare this result with that in part a).

c) If the collision time (The time it takes the cars to come to rest relative to one another) was 0.5 sec, what was the impulse given Car 1 by Car 2 and vice versa.

Answer

a)



$(M_1\ V_1) = 2800$ slug-ft/sec

$(M_2\ V_2) = 3400$ slug-ft/sec

b) In the same direction as the total momentum in (a).

Tan $\theta = 2800/3400 = .82$

c) $\vec{I}_1 = M_1 \Delta \vec{V}_1$

$$I_{1x} = M_1 \Delta V_{1x},$$

$$\Delta V_{1x} = V_{1x}' - V_1$$

$$V_{1x}' = -\frac{M_1 V_1}{M_1 + M_2}$$

(prime indicates property after collision)

$$\therefore I_{1x} = M_1 \left(\frac{-M_1 V_1}{M_1 + M_2} + V_1\right) = M_1 V_1 \left(1 - \frac{M_1}{M_1 + M_2}\right)$$

$$V_1 = -30 \text{ mph} = -45 \text{ ft/sec}$$

$$I_{1x} = 2800 \left(1 - \frac{20}{35}\right) = 400 \times 3 = 1200 \text{ lb-sec}$$

$$I_{1y} = M_1 V_{1y}' = M_1 \left(\frac{M_2 V_2}{M_1 + M_2}\right) = (20/35)(3400) = 1900 \text{ lb-sec}$$

$$\vec{I}_2 = -\vec{I}_1$$

*Note*_________This is just a result of Newton's action and reaction law.

2. Two cars, weighing 2500 lbs and 3000 lbs are moving at 30 mph and 50 mph respectively. The latter car hits the rear end of the former car and their bumpers lock.

   a) What is the velocity of the two car combination just after the collision?

   b) What is the impulse given to a 160 lb. man riding in the 50 mph car if the collision time is 1/10 sec. What is the average force acting on the man?

60

Answer

a) $M_1 V_1 + M_2 V_2 = (M_1 + M_2) V_F$

$V_F = 41$ mph

b) $I = (M_{MAN} V_F - M_{MAN} V_o)$ (I is the impulse, V_o is the initial velocity, V_F is the final velocity and M_{MAN} is the mass of the man)

$V_F = 41 \text{ mph} \approx 61 \text{ ft/sec}$

$V_o = 50 \text{ mph} \approx 75 \text{ ft/sec}$

$M_{MAN} = 160 \text{ lbs}/32 \text{ ft/sec}^2 = 5 \text{ slugs}$

$I = 5 \text{ slug } (61 \text{ ft/sec} - 75 \text{ ft/sec}) = -70 \text{ lb-sec}$

Note that the man is "thrown" forward in his car.

$F_{AVE} = \text{Impulse}/\Delta t = -700 \text{ lbs}$

3. Calculate the momentum associated with the following situations.

a) A 5 oz. baseball traveling at 100 ft/sec.

b) A 2 lb. cannon ball fired with a muzzle velocity of 500 ft/sec.

c) A 180 lb. football player sprinting at 15 mph.

d) A 1600 lb. car traveling at 10 mph: at 20 mph: at 40 mph.

e) A 180 lb. passenger in the car of part (d).

Answer

$p = MV =$ momentum

a) $p = .98$ lb-sec

b) $p = 31$ lb-sec

c) $p = 126$ lb-sec

d) $p = 750$ lb-sec
$p = 1500$ lb-sec
$p = 3000$ lb-sec

e) 84 lb-sec, 16C lb-sec, 336 lb-sec

Comment:

In bringing the above objects to a halt, the momenta of the objects must be dissipated. Forces dissipate momentum by the relation $F_{AVE} \; \Delta t = \Delta p$. Hence, if the time of dissipation (Δt) is small the forces involved will be very large. To minimize the damaging effects of large forces one should attempt to lengthen the time of dissipation. This is another good reason for wearing seatbelts in a car.

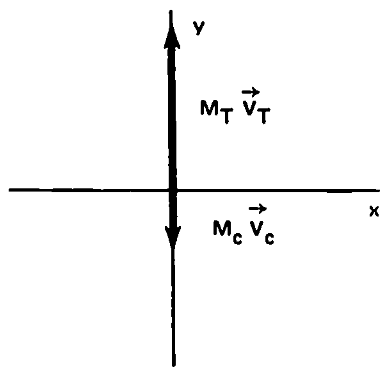
4. A truck weighing 10 tons hits a car which weighs 1 ton in a head-on collision.

   a) If the speed of the truck was 10 mph and the speed of the car was 15 mph before the impact, what is the final velocity of the two if they lock together during the collision. (Give both magnitude and direction).

   b) What is the impulse given to a 160 lb. seat belted passenger in the car? If the passenger did not wear a seat belt, in what direction — relative to the car seat — would he be thrown?

Answer

a) $M_T V_T + M_c V_c = (M_T + M_c) V_F$



$V_F = 8.75$ ft/sec in + y direction.

b) $I = M_P V_F - M_P V_o$

$M_P = 160 \text{ lbs}/32 \text{ ft/sec}^2 = 5$ slug

$I = 5 \text{ slug } (8.75 \text{ ft/sec} + 22.5 \text{ ft/sec}) = 156.3$ lb-sec

He would be thrown in the direction of his initial velocity ($-y$)

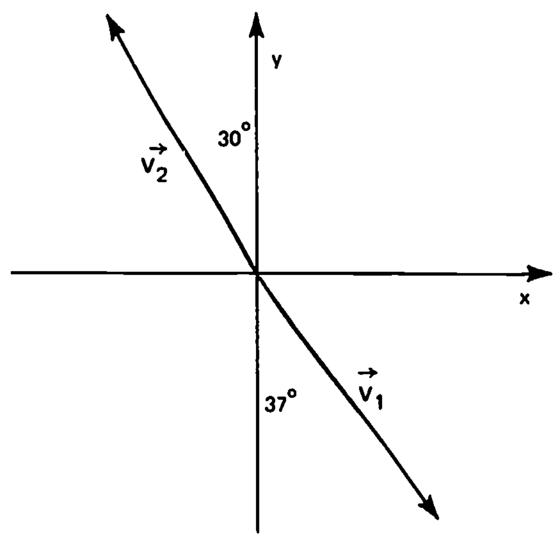
Comment:

Seat belts can be very useful in circumstances like those discussed in this problem because they minimize contact injuries as well as help a driver maintain control of his car after the collision.

5. Two cars of the same mass each traveling at 20 mph collide according to the accompanying diagram which represents the conditions just before the impact.

   a) What is the final velocity of the 2 car combination immediately after the collision if the two cars are locked together?

   b) If neither driver is wearing a seat belt, in which direction (relative to his car seat) will each driver be thrown?

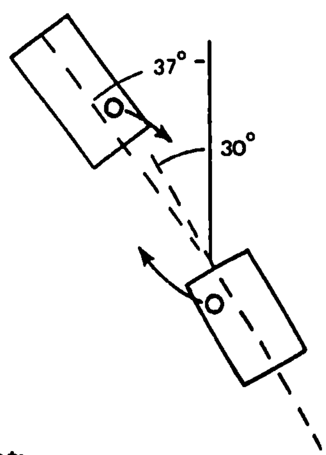


Answer

a) $V_{FX} = (V_{1x} + V_{2x})/2 = 1$ ft/sec

$V_{FY} = (V_{1y} + V_{2y})/2 = .66$ ft/sec

b)



The drivers will be "thrown" as shown by the arrow. Incidently, generally passengers are not "thrown" from or around a stopping car. The passenger has a velocity. When the car stops the passenger maintains most of his velocity (except if he wears seat belts) and it is this residual-passenger motion which is responsible for the idea of being "thrown".

Comment:

Seat belts can be very helpful in circumstances like those discussed in this problem. First they help greatly in minimizing injury due to large changes in body momentum over short time periods (high forces) and they help the driver to maintain control of his car after the collision.

Collisions and Conservation Momentum

LABORATORY #2

I. PURPOSE

A. To show the law of conservation of momentum and suggest its extension to car-car collisions.

B. To model automobile collisions and to show how to minimize one's chances cf injury in such a situation.

II. EQUIPMENT

airtrack – cart having a flat surface on which is mounted a holder to hold the objects of varying cross section

puddy

plastic container the size of one ice cube holder with one side cut out

objects of varying cross section

III. PROCEDURE

A. Using the spark apparatus of the air track:

(1) Show that momentum is conserved in a cart-cart collision where both carts are moving in the same direction.

(2) Show that momentum is conserved in a cart-cart collision where the carts are moving in opposite directions.

(3) Estimating the collision time between carts to be roughly .05 sec, calculate average forces exerted on each cart in the collisions.

B. Using an air cart with a flat plate and holder on it, mount objects of varying cross-sections and puddy holder. Allow cart to collide with a barrier so that puddy holder slams against the striking object. For each striking object used, measure the effective cross-sectional area and depth of penetration into the puddy. Measure cart velocities with the spark apparatus and try to run a whole set of these puncture experiments at the same velocity. (A stop watch may be useful for estimating the velocity.)

IV. QUESTIONS

A. Is momentum conserved in a car-car collision?
Is energy conserved in a car-car collision? Explain your answers.

B. Considering part IIIB, what parallel can you make between the experimental results and a car-car collision? (constrained versus unrestrained passenger)

C. What is the most desirable distribution of force in a car accident? How can you best arrange to get this distribution?

V. COMMENTS

A. It should be pointed out to the students that the physical laws describing the collision between two objects such as two cars or a car and its passengers are the same laws that govern the collision between two carts. The Law of Conservation of Momentum works equally well when two cars are concerned as when two carts are used. Of course, there may be external friction forces present when the two cars collide which would cause the momentum conservation law to be violated.

B. The object of the collision of something soft with a hard object of varying cross section is to demonstrate:

   a) The collision of an unseatbelted passenger with various parts of the car.

   b) Deformation (strain) is directly related to stress. Hence by spreading the same force over a larger area the deformation is reduced. In a car collision bodily injury will be reduced if seat belts are used.

REVIEW

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MOMENTUM AND IMPULSE

Review

Concept Review

1. **Momentum** — If an object of mass M is moving with a velocity V along a straight line in the positive direction, then its momentum is $p = MV$.

2. **Impulse** — The impulse between two times is just the change in the momentum between those two things. Hence, if the momentum is measured at time t and $t + \Delta t$ then $\Delta p = p(t + \Delta t) - p(t) = M\Delta V$ = impulse given to the object between t and $t + \Delta t$.

3. **Impulse and average force** — The average force F_{AVE} exerted on a body through a time interval Δt is $F_{AVE} = M\Delta V/\Delta t = \Delta p/\Delta t$. Hence, $F_{AVE}\Delta t = \Delta p$ = impulse.

4. **Law of conservation of Momentum (Single body)** — The momentum of a body will remain constant providing no net (resultant) force acts on the body.

5. **Momentum of a system of objects** — If you consider a system composed of more than one object the momentum of the whole system is just the vector sum of the momenta of the objects composing the system.

QUESTION

Problem Review

1. A car weighing 1 ton moves down a long straight road at 30 mph.

    a) What is the momentum of the car?

    b) What impulse is necessary to slow the car to 10 mph?

    c) If the transition to 10 mph occurs in 5 seconds, what average force was applied to the car?

    d) If a 160 lb. man were in the car of part (c) above, roughly what force would his seat belt exert on him?

Answer 1

a) $p = MV$

$M = (2000 \text{ lbs})/(32 \text{ ft/sec}^2) = 62.5 \text{ slugs}$

$V = 30 \text{ mph} = 45 \text{ ft/sec}$

$p = 62.5 \text{ slugs} \times 45 \text{ ft/sec} = 2812.5 \text{ lb-sec}$

b) From part (a) above $p = -2812.5$ lb-sec

If car slows to 10 mph, then $p_{Final} = 1/3\, p_{Initial}$

Then the impulse is $p_{Final} - p_{Initial} = 1/3\, p_{Initial} - p_{Initial} = 2/3\, p_{Initial} = -1875$ lb-sec that is the momentum change is in the opposite direction as the velocity of the car.

c) $F_{AVE} = \Delta p/\Delta t = -2812.5/5 \text{ sec} = -562.5 \text{ lbs.}$

d) $F_{AVE} = \Delta p/\Delta t = \frac{2}{3}\, p_{initial}/5 \text{ sec}$

$p_{initial} = 5 \text{ slugs} \times 45 \text{ ft/sec} = 225 \text{ lb-sec}$

$F_{AVE} = 30 \text{ lbs}$

SOMETHING TO THINK ABOUT

This problem (part d) shows a situation where almost no one would be injured by the forces involved, however a seat belt would eliminate the annoying small-movement response to the 30 lb. force.

2. A bullet weighing 1/3 of an ounce is fired with a muzzle velocity of 1000 ft/sec.

   a) What is the momentum?

   b) If the bullet stops in .0005 sec what amount of average force is required?

   c) A baseball weighing 5 ounces is thrown at a speed of 100 ft/sec. What is the momentum of the baseball?

   d) What average force is required to stop the ball in 1/20 sec?

   e) Why will the bullet in general do more damage to an object than will the baseball?

   f) A 160 lb. passenger is in a car moving at 20 mph. Calculate the momentum of the passenger.

   g) What average force is necessary to stop the passenger in 1/10 sec?

Answer 2

a) $p = MV$

$$M = \frac{(1/3 \text{ oz})\ (1/16 \text{ lbs/oz})}{32 \text{ ft/sec}} = 1/1536 \text{ slugs}$$

$V = 1000$ ft/sec

$p = 1.30$ lb-sec

b) $F_{AVE} = \Delta p/\Delta t = (0 - 1.30 \text{ lb-sec})/.0005 \text{ sec} = -2600$ lbs
(The minus sign says that the force must be applied in a direction opposite to the bullet's momentum.)

c) For the baseball
$p = MV$
$M = (5/16)\ (1/32)$ slugs = .00978 slug
$V = 100$ ft/sec
$p = .978$ lb-sec

d) $F_{AVE} = \Delta p/\Delta t = -.978 \text{ 16-sec}/1/20 \text{ sec} = -19.6$ lbs

e) Because the bullet requires a large force to stop it and because the crossectional area of the bullet is very small as compared with the baseball.

f) For the passenger in the car
$p = MV$
$M = 160 \text{ lbs}/32 = 5$ slugs
$V = 20$ mph = 30 ft/sec
$p = 150$ lb-sec

g) $F_{AVE} = (0 - 150 \text{ lb-sec})/1/10 \text{ sec} = -1500$ lbs.

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3. Two cars both weighing 1.6 ton are moving along a straight road, the first at 20 mph and the second at 30 mph.

   a) What is the total momentum of the system of two cars if they are moving in the same direction?

   b) If the cars were to collide head on, what would be the total momentum of the cars just after the collision? Assume that the momentum of the 30 mph car is positive.

Answer 3

a) Total momentum = $p_1 + p_2$

1 corresponds to the 20 mph car

p_1 = MV = (3200 lbs/32 ft/sec^2) x 30 ft/sec = 3000 lb-sec

2 corresponds to the 30 mph car

p_2 = MV = (3200 lbs/32 ft/sec^2) x 45 ft/sec = 4500 lb-sec

$p_{TOTAL} = p_1 + p_2$ = 7500 lb-sec

b) Since total momentum of the two cars together is conserved in the collision, the total momentum before the collision is the same as the total momentum after the collision.

Hence, $p_{TOTAL} = p_1 + p_2$

If p_2 = 4500 lb-sec is a positive quantity, then since the collision is head-on, the other car must be moving in the negative direction. Hence p_1 = −3000 lb-sec

p_{TOTAL} = 4500 lb-sec − 3000 lb-sec = 1500 lb-sec

FORCE

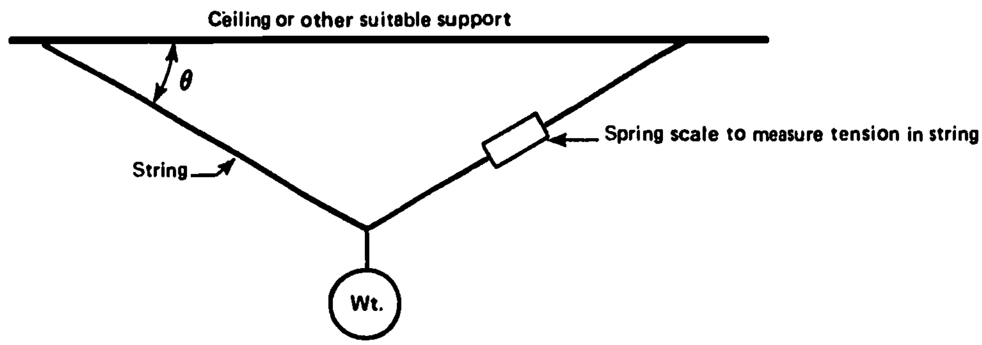
Demonstrations

Demonstration 1

Connect two or more strings weighted via pulleys to an object and change the angles of the various strings until an equilibrium position for the object is reached. Then show that components of the forces (represented in direction by the strings) add to zero when all strings are considered.

Demonstration 2

Hang weights using the following set up.



Determine how far a given weight the tension in the string depends on θ. (If you can put the weight shown above on a pulley, then *both* acute angles will be θ.)

Demonstration 3

Using the rubber band stop technique, determine the maximum force used in stopping cars of varying momentums.

FORCE

Examples and Problems

1. A man, weighing 160 lbs., drives a car around an unbanked circular turn of radius 500 ft. If the car is moving at 20 mph:

   a) What force must be supplied to keep the passenger in his seat?

   b) Suppose the car is moving at 40 mph, what force is required?

Answer

a) $F = M\,a \qquad a = V^2/R \qquad F = 9\ \text{lb}$

b) $F \propto V^2$

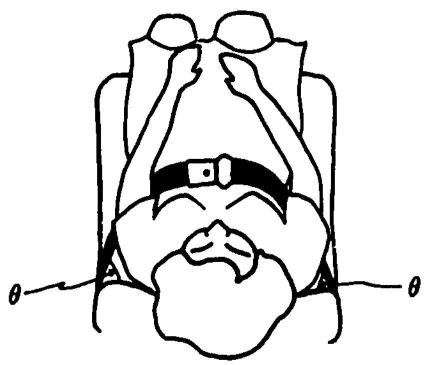
$\therefore F = 36\ \text{lb}$

2. During a collision a woman who weighs 128 lbs is held in place by a seat belt. If the car stops from 40 mph in 0.1 sec:

   a) What is the average force a seat belt must apply to hold the woman in place?

   b) If the seat belt is as shown below, what average force must the seat belt be able to withstand?

$\theta = 60^\circ$



Answer

a) $a_{AVE} = \Delta V/\Delta t = -60\ (\text{ft/sec})/.1\ \text{sec}$

$F_{AVE} = M\,a_{AVE} = 2400\ \text{lbs}$

b) $F_{SB} \doteq 2770\ \text{lb}$ (tension)

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3. A sports car traveling at 20 mph makes a 90° circular turn with a radius of 20 ft. (This approximates the car moving around a sharp corner). Assuming the driver weighs 160 lbs, what is the maximum force that a seat belt must supply to hold him in position? (Neglect the friction between the man and the seat.)

Answer

$F = M\,V^2/R = 225$ lb. towards the inside of the turn.

4. A car weighing 3000 lb rounds a level curve of radius 400 ft. The road is unbanked and the velocity of the car is 60 mph.

   a) What is the minimum coefficient of friction so that the car will not skid?

   b) What is the force a seat belt must apply to hold the driver in place if we assume the coefficient of friction between the driver and the seat is ½ that between the road and the car. (The driver weighs 192 lbs.)

Answer

a) $\mu F_N = M\,V^2/R$
$F_N = 3000$ lbs
$\mu = 0.6$

b) $F = 58$ lbs

5. On dry concrete a car weighing 4000 lbs and traveling at 60 mph can stop in 366 ft.

   a) What is the average force which the concrete exerts on the wheels?

   b) What force does a 200 lb passenger feel?

   c) In what direction will the passenger in part b move (relative to the car) if he is not restrained by a seat belt? Doesn't this violate one of Newton's Laws?

Answer

a) $2\,ax = V_F{}^2 - V_0{}^2$
$F = M\,a$
$F = -1320$ lbs.

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b) $F = -66$ lbs. (Minus sign means that the force is opposite to the original direction of motion for the car.)

c) The Passenger will move forward relative to the car. If he was initially at rest relative to the car and he starts to move relative to the car, then clearly a force is at work in the direction of his motion, since we know that $F = M\,a$ – right? wrong! The car is an accelerated reference system, and $F = M\,a_{ReV}$ where F is a *physical force,* does not hold. What really happens is that the car decelerated leaving the passenger to fend for himself. The passenger maintains his previous velocity while the car is literally stopping around him.

6. A 2 lb book is placed on the level front seat of a car. If the coefficient of friction is $\mu = 0.5$, what minimum acceleration must be exceeded for the book to move toward the back of the seat? If the book is being accelerated forward, why does it move backward?

Answer

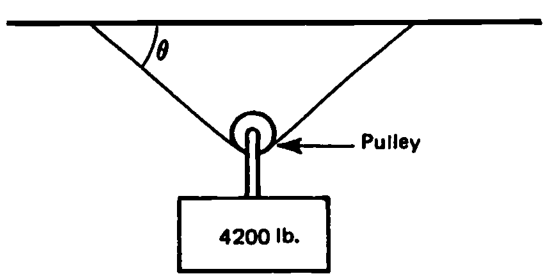
$F_{Fr} = \mu F_N = M\,a$ (F_{Fr} = Friction force; F_N = normal force)

$F_N = M\,g$

$\mu M\,g = M\,a$

$a = \mu\,g = 0.5\,g$

7. In a car to car collision at 45 mph the peak seat belt load is 4200 lbs. Suppose that a seat belt will break if subjected to a tension exceeding 5000 lbs. To model the seat belt during the collision situation consider the following arrangement.



a) At what angle θ_c will the seat belt just support the 4200 lbs?

b) Suppose $\theta < \theta_c$ what will happen?

c) Suppose $\theta > \theta_c$ what will happen?

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Answer

a) $T \sin \theta = 4200$ lbs. If $T = 5000$ lbs. (Breaking point), $\sin \theta = .84$, and $\theta = 57$

b) The belt will break.

c) The belt will not break.

8. In a certain accident a seat belt was used and the belt assembly remained in tact during the accident. Later laboratory tests showed that the belt assembly would fail when the seat belt was stressed uniformly by 5400 lbs (400 lbs above legal minimum.) A policeman said he clocked the driver's car just prior to the accident and found the car to be going 60 mph in a 50 mph zone. Hence, a ticket was to be issued to the driver. The driver weighing 200 lbs., went to court and did not have to pay the ticket. Why not? (Only physics answers are acceptible.) Assume actual accident was about head-on and the collision time was 1/10 sec.

Answer

$F_{AVE} = M\, a_{AVE}$; $a_{AVE} = \Delta V/\Delta t$; assume $\Delta V = -60$ mph $= -88$ ft/sec

$t = 0.1$ sec, $M = 200 \text{ lb}/32 \text{ ft/sec}^2$

$F = -5500$ lbs.

The seat belt would have broken in the accident if the man had been driving 60 mph.

Comment:

Actually in an accident of this sort the peak force is 2 to 3 times the average force, therefore, the man was probably going a bit slower than 50 mph.

9. A car moving at 30 mph collides with a wall. The belted driver decelerates with the car and essentially comes to rest after 1/10 sec. The head of an unrestrained passenger, however, hits the dashboard and comes to rest in about 1/60 sec after initial dashboard contact. If the riders in each case weigh 160 lbs. and the passenger's head weighs 20 lbs., determine the average force each feels.

Answer

$F_{AVE} = M\, \Delta V/\Delta t$

$\Delta V = 0 - 45 \text{ ft/sec} = -45 \text{ ft/sec}$

For seat belted driver

$M = 160 \text{ lbs}/32 \text{ ft/sec}^2 = 5 \text{ slugs}$

$\Delta t = 0.1 \text{ sec}$

hence, $F_{AVE} = 5 \times (-45)/0.1 \text{ lbs} = -2250 \text{ lbs.}$

For unrestrained passenger

$M = 20 \text{ lbs}/32 \text{ ft/sec}^2 = 5/8 \text{ slug}$

$\Delta t = .02 \text{ sec}$

$F_{AVE} = 5 \times (-45)/[(8)(.01)] = -2812 \text{ lb.}$

Comment:

Note that the forces in the two cases are comparable but for the unrestrained passenger the force is applied to the *head alone.*

A Collision Model

LABORATORY

I. PURPOSE

To analyze a simple model for a collision in terms of the physical properties involved (peak force, average force, collision distances and times), and to get a rough estimate of collision times in car accidents.

II. EQUIPMENT

air track with spark apparatus

5-6 rubber bands

scale to weigh objects — the carts

spring balance

III. PROCEDURE

1. By holding a single rubber band across the track of an air cart (at a pre-recorded position) reflect (i.e., bounce back) an incoming cart with the sparker in operation. The position of the cart will be recorded at a number of times (use the 60 Hz sparker). From this position versus time information, one can obtain the initial cart velocity. The collision distance and the collision time. By including the mass of the cart (by weighing) one can also obtain the initial cart momentum and the impulse and average force applied to the cart during the collision process.

   a. Record collision time

   b. Record momentum, impulse and average force

   c. Record maximum force. (This can be determined by recording the displacement of a spring balance when attached to the rubber band which is then forced to extend to the collision distance. See figure.)

   Plot maximum force versus inverse collision time.

2. Using first one rubber band, then two, then three and so on, repeat the experiment in part (1) above with the velocity of the cart held roughly constant. For each run record the maximum force and the collision time and the collision distance. Plot maximum force versus inverse collision time.

   Also plot maximum force versus inverse collision distance.

IV. QUESTIONS

A. For barrier collision involving a car traveling at a speed between 20 mph and 50 mph, the collision time (time from initial contact to final stopping) is about 1/10 sec. How does your collision time in part (1) of this experiment compare with this?

B. Using part A compare the average force with the maximum force. Sketch a rough graph of the force exerted on the cart by the rubber band versus time and discuss how the average force might be obtained.

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C. In a car collision at 30 mph with a barrier, a seat belted passenger stops in a distance of about 2 ft. and in a time of 1/10 sec. In the same collision an unrestrained passenger may have a stopping distance of several inches in a time of 1/100 sec. From part B of this experiment, what would you conclude about the relative amounts of force on the two passengers?

V. COMMENTS

A. The student can also measure the force as a function of time by using the method of part IIIa at those positions where the spark timer discharges. By plotting the force versus time curve the impulse (area under the force vs. time curve) given to the car may be compared to the change in momentum.

B. It may also be pointed out to the student that on a basis of what he learned about the relationship of the maximum force to the collision time, one can see why the deceleration time during a collision should be maximized. A very good way to do this is to "ride" the relatively slow crushing of the car (rather than to be thrown about the passenger compartment and undergo very sharp decelerations). The student should realize that a seat belt will enable him to do just that.

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FORCE

Review

Concept Review

1. **Isolated object** — An object is isolated if it is entirely unaffected by its surroundings.

2. Newton's Laws —

   1. If a body is isolated, then the velocity of that body remains unchanged according to a non-accelerating observer.

   2. If a body changes its velocity, then a force is said to be acting on the body. Hence, for a body moving along a straight line:

      $$F = Ma$$

      where F = variable standing for the force

      M = mass of the body

      a = acceleration of the body.

   3. If one body exerts a force on another body, then the latter body exerts an equal — in-magnitude — but opposite-in-direction force on the first.

3. Average Force — $F_{AVE} = M\, a_{AVE} = M \Delta V/\Delta t$

4. **Weight** — The weight of an object is the amount of force needed to support an object when it is acted on only by gravity. **w** = Mg where M = mass and g = acceleration due to gravity.

QUESTION

Problem Review

1. A young lady weighs 128 lbs.

   a) What is her mass in slugs, in kg, in gms.?

   b) If she is accelerated in a car by 1/8 g what force is being exerted on her?

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Answer 1

a) $W = Mg$ so $M = w/g$

$M = 128 \text{ lbs}/32 \text{ ft/sec}^2 = 4$ slugs

1 slug = 14.6 kg and 1 kg = 1000 gM

b) Force = weight x acceleration in g units

= 128 lbs x 1/8 = 16 lbs.

2. If a boy rides a bike weighing 16 lbs and causes the bike to accelerate at 2 ft/sec^2, what is the magnitude of force exerted on the combination – boy + bike? By what is this force exerted? How does this force come into existence? The boy weighs 128 lbs. (An acceleration of 2 ft/sec^2 will give the boy + bike a velocity of 15 mph in about 11 sec if the combination starts from rest.)

Answer 2

$F = M_{Total}a$

$M_{Total} = M_{Bike} + M_{Boy} = 16\ lbs/32\ (ft/sec^2) + 128\ lb/32\ (ft/sec^2) = 4.5$ slugs

$a = 2\ ft/sec^2$

$F = 9$ lbs.

Road exerts force in reaction to riders force communicated through the tires.

SOMETHING TO THINK ABOUT

It has been said in jest that bicycle and motorcycle riders should wear seat belts. Good idea or bad? Explain, using some physics ideas.

3. In an emergency, a 160 lb. man is forced to jump from a 16 ft. high wall onto a cement sidewalk. If his time to stop after hitting the sidewalk is roughly 1/10 sec, determine:

   a) The velocity of the man just before he hits the sidewalk.

   b) The average force exerted on the man during the jump (before he hits).

   c) The average force exerted on the man during his collision with the sidewalk.

Answer 3

a) $V = gT$ = velocity just before impact.

$g = 32\ ft/sec^2$

T = time it takes for a man to fall

We note that he fell 16 ft., and that fall distance = $16\ T^2$ so that $T = 1$ sec.

Hence $V = 32$ ft/sec = 22 mph

b) When gravity acts alone, the force on the man is his weight. (We neglect Air Friction.)

c) $F_{AVE} = M\ \Delta\ V/\Delta\ t$

$M = 160\ lbs/32\ ft/sec^2 = 5$ slugs

$\Delta\ V = -32$ ft/sec

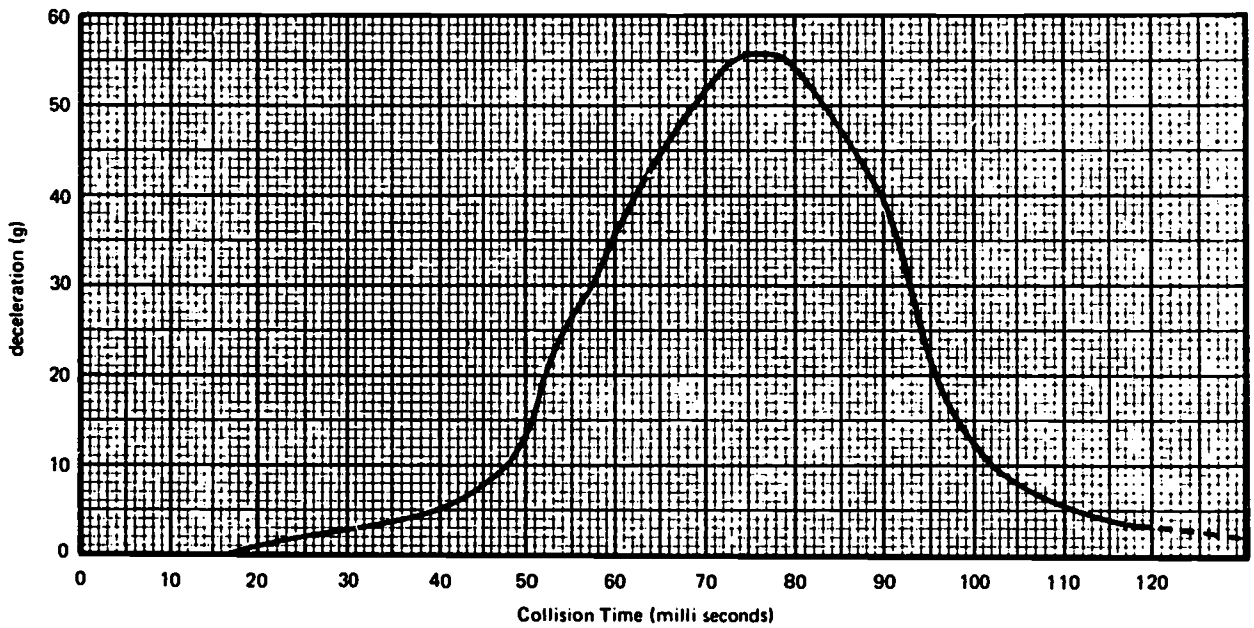
$t = 1/10$ sec

$F_{AVE} = -1600$ lbs. (upwards)

SOMETHING TO THINK ABOUT

The average forces in many automobile head-on collisions may exceed the force value calculated in 3c — even for seat belted passengers. For unrestrained passengers the average force is much greater.

4. In a head-on collision at 33 mph with a barrier, the deceleration of a seat belted passenger looks as follows:



a) Draw a graph of force on the passenger *vs* time if the passenger is a young boy weighing 64 lbs.

b) What is the maximum force exerted on the boy?

c) What is the average force exerted on the passenger over the period 20 m sec to 120 m sec? (A m sec is one 1/1000 sec.) Hint: Find the height f of the bar graph shown in second figure such that f x T = f x 100 m sec = area under f(t) *vs* t curve from t = 20 m sec to t = 120 m sec = area under f(t) *vs* t curve from t = 20 m sec to t = 120 m sec drawn in part (a).

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Answer 4

a) The graph has the same shape but the vertical height is multiplied by 64/32 slugs = 2 slugs

b) The maximum force exerted on the boy is 55g x 64 lbs = 3520 lbs.

c) Guess about 21.50 squares for area

1 square = 10 g x 10 m sec = 100 g – m sec (on acceleration graph)

= 6400 16 – m sec (on force graph)

So total area under f(t) *vs* t curve is 1.376×10^5 lb-sec with $\Delta t = 100$ m sec.

Then $F = F_{AVE} = 1.376 \times 10^3$ lbs = 1376 lbs.

SOMETHING TO THINK ABOUT

Compare the average force in 4c with the average force exerted on the man in 3c.

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5. In a car test with an unrestrained dummy (no seat belt), after the car moving at 40 mph collides with a wall, the dummy is thrown forward and his head hits the windshield. The time it takes the dummy's head to come to rest, roughly 1/100 sec, is much less than that for the car to come to rest, about 1/10 sec. If just before his head hits, the dummy is "moving" at 40 mph and his head weighs about 20 lbs:

   a) What average force is exerted on the dummy's head during the collision?

   b) Draw a rough graph of this force against time and compare the graph with the graph in problem 2. (Consider only the magnitude) (We have assumed that the average force and the peak force are about the same for the dummy's head.)

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Answer 5

a) $F_{AVE} = M \Delta V/\Delta t$

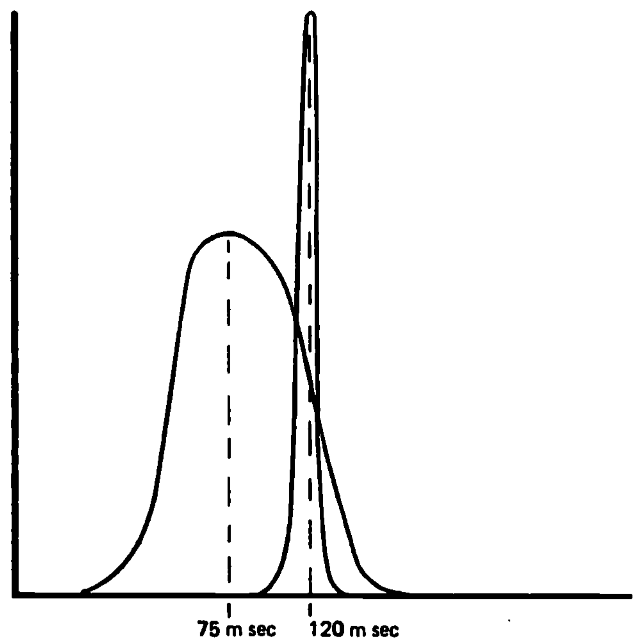
$M = 20 \text{ lbs}/32 \text{ ft/sec}^2 = 5/8 \text{ slug}$

$\Delta V = -40 \text{ mph} = -60 \text{ ft/sec}$

$t = 1/100 \text{ sec}$

$F_{AVE} = -5/8 \times 60 \times 100 \text{ lbs} = -3750 \text{ lbs.}$

b)



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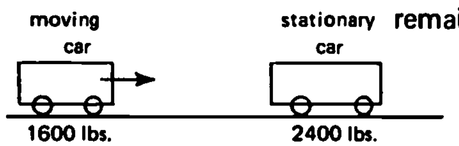
6. Two cars collide head to tail over a time interval of 1/10 sec. The car in front, weighing 2400 lbs. is unbraked but stationary. The car behind, weighing 1600 lbs. is moving in the direction shown at 20 mph. If the cars remain hooked together after the collision and move at 8 mph:



a) What are the magnitude and direction of the average forces exerted on the cars?

b) Explain this in light of Newton's action and reaction law.

c) What force was exerted on the cars?

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Answer 6

a) Car behind $F = M \Delta V / \Delta t$

$M = 1600 \text{ lbs}/32 \text{ (ft/sec}^2\text{)} = -18 \text{ ft/sec}$

$\Delta V = 12 \text{ ft/sec} - 30 \text{ ft/sec} = -18 \text{ ft/sec}$

$\Delta t = 1/10 \text{ sec}$

$F_{AVE} = -50 \times 18 \times 10 = -9000 \text{ lbs.}$

The minus sign means that the force is opposite to the direction that the car was initially moving.

Car ahead $M \Delta V / \Delta t$

$M = 2400 \text{ lbs}/32 \text{ ft/sec}^2 = 75 \text{ slugs}$

$\Delta V = 12 \text{ ft/sec} - 0 = 12 \text{ ft/sec}$

$\Delta t = 1/10$

$F_{AVE} = 75 \text{ slugs} \times 12 \text{ ft/sec} \times 10 \text{ sec} = 9000 \text{ lbs.}$

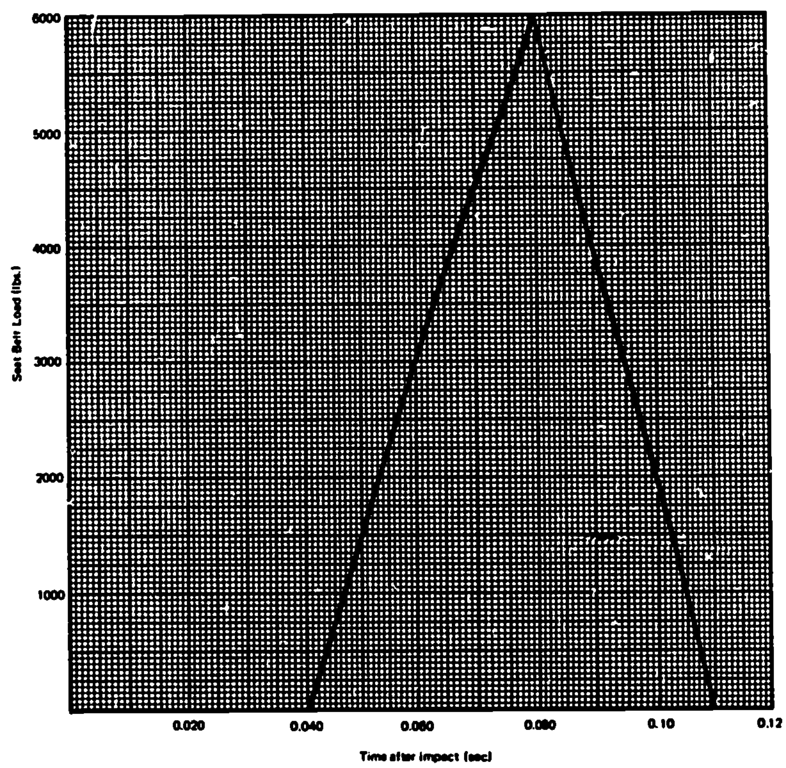
b) This result is a simple example of Newton's action and reaction law.

c) The total force exerted on the cars is zero because when considered together as a system all internal forces cancel and we have neglected any external forces like road friction.

IMPULSE

Examples and Problems

1. In a head-on collision with a brick wall the seat belt loading on a passenger in the front seat looks very roughly as follows:



   a) What is the total impulse given to the 120 lb. boy?

   b) What was the speed of the car before contact with the wall? (Assume that the final speed is zero.)

Answer

a) impulse = area under curve
   = 210 lb-sec

b) $M \Delta V = 210$ lb-sec

   $M = 120$ lbs/32 ft/sec^2 = 3.75 slugs

   $\Delta V = 56$ ft/sec

   $\Delta V = V_F - V_O; V_O = o$

   $V_F = 56$ ft/sec ≈ 38 mph

2. Let us compare the average amount of force acting on a person when he is hit in the chest by a baseball and when he is stopped by a steering column in a 20 mph head-on collision.

   a) A baseball weighing 5 oz. and thrown by a major league pitcher often travels about 100 ft/sec. If the baseball comes to rest, after hitting the batter, in approximately 0.050 sec., calculate the average force that the batter feels.

   b) Suppose that the car travels at 20 mph and the effective weight of the driver hitting the steering column is 90 lbs. If the duration of the force is 0.1 sec., what is the average force against the man's chest?

   c) To properly compare the effect of the forces in (a) and (b), we should also consider the effective area over which these forces are applied. We will consider the force due to the baseball as applied over 1 square inch and that due to the steering column as applied over 10 sq. inches. Now compare the effective "pressures" or force per unit area.

Answer

a) $\Delta (MV) = M (V_F - V_o) = F \Delta t$

$M = (5/(16 \times 32))$ slug

$V_o = 100$ ft/sec, $V_F = 0$, $t = .05$ sec

$F = -20$ lb.

b) 20 mph $= 30$ ft/sec $= V_o$, $V_F = 0$

$\Delta t = 0.1$ sec, $M = 90/32$ slugs

$F = M \Delta v / \Delta t = -840$ lb.

c) $P_{BB} = 20$ lb/in^2

$P_{Sc} = 84$ lb/in^2

Comment:

When a belted driver hits the steering column, his chest will tend to be parallel with the plane of the wheel. Hence the Force in 2(b) will be distributed over a larger area than that of the unrestrained driver whose lower chest hits just the lower part of the wheel. To reduce injury due to a given Force, generally the larger the area of application the less the injury.

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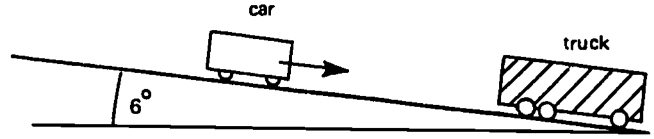
3. While waiting at a traffic light a driver begins to daydream and unknowingly releases the foot brake on his car. His car moves a distance of 5 ft down a grade which makes an angle of 6° with the horizontal and hits a stopped truck.

   a) With what speed does he hit the truck?

   b) What is the impulse given the driver of the car when the car is stopped by the truck? (The man weighs 160 lbs.)

   c) What average force is exerted on the car if the stopping time is 0.1 sec? (The car weighs 3200 lbs.)

   d) What average force is exerted on the driver in part (b) if he wears a seat belt and hence stops with the car?



Answer

a) $2ax = V_F^{\,2} - V_0^{\,2}$
   $a = g \sin 6°$
   $V = 5.5$ ft/sec

b) $I = M \Delta V = 27.5$ lb·sec

c) $F_{AVE} = 5500$ lbs.

d) $F_{AVE} = \dfrac{M \Delta V}{\Delta t} = 275$ lb

4. An unrestrained passenger in the front seat of a car which is involved in a head-on collision at 20 mph is thrown forward and then stopped by hitting the dashboard and windshield.

   a) If the windshield stops the man's head in about 0.01 sec. what is the average force exerted on the man's head if its effective weight is 20 lbs.?

   b) If the man's chest which effectively weighs 90 lbs, is stopped by contacting the dashboard and the stopping time is 0.05 sec, what is the force applied to the man's chest?

Answer

a) $F_{AVE} \Delta t = M \Delta V$
   $\Delta t = 0.01$, $M = 20/32$ slug. $\Delta v = -20$ mph $= -30$ ft/sec
   $F_{AVE} = -1875$ lbs.

b) $F_{AVE} = -1687$ lbs.
   (What is the meaning of the minus sign in the above answers?)

5. A 96 lb. boy jumps off a 16 ft. retaining wall and lands on the concrete sidewalk below.

   a) Find the boy's velocity just before he contacts the sidewalk.

   b) Find the momentum of the boy just before contact.

   c) Find the impulse.

   d) Find the average force if the boy comes to rest in 0.1 sec.

   e) Find the boy's average deceleration.

Answer

a) $V^2 = 2\,a\,x$
   $V = 32$ ft/sec

b) $p = 96/32 \times 32$ ft/sec $= 96$ lb-sec $(P = mv)$

c) $\Delta\,(MV) = 96$ lb-sec

d) $F = -960$ lbs.

e) $a = 320$ ft/sec $= 10\,g$

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TORQUE

Examples and Problems

1. What is the effective torque about the point midway between a man's shoulders and head when his car is hit in the rear end? The effective acceleration of his car is 10 g's. Consider only the man's head so that in fact we are calculating the torque responsible for "whiplash". His head is assumed to weigh 20 lbs. and may be considered located at its actual center of mass, 9" from the pivotal point about which we want the torque.

Answer

torque = R x F = 150 lb ft

2. A baseball thrown by a pitcher at 100'/sec is hit directly back at the pitcher with the same speed. If the impact time was 0.1 sec for the ball hitting the bat, and the ball weighs 5 oz.:

   a) What is the average torque exerted by the ball on the bat with respect to a point on the bat just between the batter's hands? This point is 30 inches from the impact point of the ball upon the bat.

   b) What is the average force on the ball?

Answer

a) $F \Delta t = M \Delta V$

$M = w/g = [5/(16 \times 32)]$ slug

$\Delta V = 200$ ft/sec, $\Delta t = 0.1$ sec

$F_{AVE} = 20$ lb

b) torque = R x F = 50 lb ft

3. To remove a wheel nut, a 90 lb boy finds he must stand on the lug wrench at a point 9 inches from the center of the nut. What is the torque in lb-ft exerted by the boy under these conditions?

Answer

torque = 68 lb-ft

ENERGY

Examples and Problems

1. Calculate the energy associated with the following situations:

    a) A baseball of weight 5 oz. traveling at 100 mph.

    b) A 200 lb. football player sprinting at 15 mph.

    c) A 160 lb. passenger in a car at 20 mph; 40 mph.

    d) The car in part c) which weighs 2000 lbs.

Answer

a) K. E. = ½ M V
K. E. = 110 ft-lb

b) K. E. = 1510 ft-lb

c) K. E. = 2250 ft-lb K. E. = 9×10^3 ft-lb

d) K. E. = 2.8200×10^4 ft-lb
K. E. = 1.13×10^5 ft-lb

Comment:

In order to stop a moving object we must somehow dissipate its kinetic energy. This is done by doing work on the object – that is letting a force act on an object through some distance. Hence, we write $F_{AVE} \Delta x = ½ M V^2$. Note that if stopping distances are small the average forces are large. Here again is a good reason to wear a seat belt since the belt will actually lengthen the collision stopping distance.

2. A car weighing 3200 lbs. is brought to rest from 30 mph in a distance of 2 ft.

    a) Assuming that a constant force acts on the car, what is the work done by this force?

    b) Determine the average force using part (a).

    c) Where does the kinetic energy of the car go?

Answer

a) W = change in Kinetic Energy = Δ K. E.
$W = -1.01 \times 10^5$ ft-lb

b) $W = F \Delta x$
$F = -5.1 \times 10^4$ lb

c) Into the heating of tire and metal

3. If a man stops his car from 40 mph to 0 mph very quickly so that we can assume the seat belt holds the man in place:

   a) How much work will the seat belt do if the man weighs 192 lb.?

   b) If the man moves 2 ft with the crushing of the car, what average force is being exerted on the man?

Answer

a) $K.E._0$ = Initial Kinetic Energy
$K.E._F$ = Final Kinetic Energy
$W = K.E._F - K.E._0 = -K.E._0 = -\frac{1}{2} M V = -1.08 \times 10^4$ ft-lb

b) $F_{AVE} = W/\Delta x = -5400$ lbs.

Comment:

The energy of the man must be dissipated by the body during the collision. By using a seat belt and harness this energy is dissipated by the displacement of fluid and organs within the body with, often times, resulting ruptures and hemorrhages. However, if a seat belt is not used, the energy is dissipated by the crushing of the body and the car as the passenger strikes relatively hard objects in the passenger compartment.

4. a) What is the momentum of a 10 ton truck traveling at 30 mph?

   b) What would the velocity of a 5 ton truck have to be in order to have the same momentum?

   c) What would the velocity of the 5 ton truck have to be in order to have the same kinetic energy as the 10 ton truck in a)?

Answer

a) $P = 2.75 \times 10^4$ lb-sec

b) $V = 60$ mph

c) $\frac{1}{2} M_5 \; V_5{}^2 = \frac{1}{2} M_{10} \; V_{10}{}^2$
$V_5{}^2 = M_{10}/M_5 \; V_{10}{}^2$
$V_5 = 42.3$ mph

5. a) Using energy consideration determine how fast a 160 lb. man would be traveling when he hits the ground if he falls from a 20 ft scaffolding.

b) What is this in mph?

c) If the man falls supine and is stopped in 0.02 sec., what is his average deceleration?

d) What is the average force on the man?

e) What is the average ground pressure on the man during the impact if his effective contact areas is sq-ft.?

f) What will be the depth of the depression made in the ground by the man?

Answer

a) $mgh = \frac{1}{2} M V^2$
$V = 36$ ft/sec

b) 24 mph

c) $a = V/T = 1800$ ft/sec^2

d) $F = Ma = 9000$ lbs.

e) $P = F/A = 3000$ lb/ft^2

f) $d = 0.36$ ft

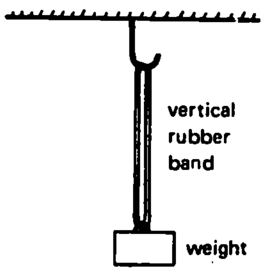
STRESS AND STRAIN

Demonstrations

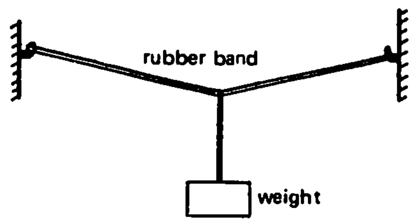
Demonstration 1

Plot a Force *vs* Strain curve for a rubber band in the two configurations shown below:

a)



b)



Why are the two curves basically different in their small load behavior?

Demonstration 2

Using level air track, push carts holding objects of varying cross section into some soft material held in a holder at the end of the track. Show that for the same cart velocity the objects of larger cross section produce less deformation. This is similar to using seat belts to spread out collision forces over a larger area, hence small damage is done to the body.

STRESS AND STRAIN

Examples and Problems

1. a) In a head-on collision at 20 mph it is found that an unrestrained passenger may bump his head against the windshield with a force of the order of 1500 lbs. If these forces are distributed over a contact area of 3 sq. inches, what is the pressure applied to the man's head?

   b) If the forces on the man's chest from the dashboard are the order of 1000 lbs. over an area of 10 sq. inches, what pressure is involved?

   c) If the man had been held in place by a seat belt, the total forces applied to the man's body would be about 2000 lbs. and these forces would be applied by the seat belt over an area 2 in. by 12 in. What would then be the pressure exerted by the seat belt on the man?

Answer

a) $P = F/A = 500\ \text{lb/in}^2$

b) $P = 100\ \text{lb/in}^2$

c) $P = 83\ \text{lb/in}^2$

Comment:

Even these numbers do not tell the whole situation. The forces in part a) are applied to the head and may result in concussions, etc. at a very vital spot. In b) the forces are upon the ribs and such forces may cause breaking. In c), however, the forces are distributed over a not so vital area of the body which contains the very strong pelvic bones. The seat belt spreads out the applied force. These results provide one of the best arguments for the use of seat belts in automobiles.

2. A car weighing 4000 lbs. is brought to a stop from 40 mph in .4 ft. during a barrier crash study.

   a) What is the average stopping force?

   b) Assuming this force acts over an area of 10 sq. ft., what is the stress on the car during the crash in lbs/in^2?

Answer

a) $F \times L = \frac{1}{2} M V$

$L = 4'$, $\dot{M} = 4000/32$ slug, $v = 40$ mph $= 60$ ft/sec

$F = 56{,}250$ lbs

b) $56{,}250\ \text{lb}/10\ \text{ft}^2 = 39\ \text{lb/in}^2$

3. Suppose that we have 3 cylinders each filled to the height of one foot with some material which has a Young's modulus of 10 newton/meter^2. The cross sectional areas of the 3 columns are 1 sq. in., 3 sq. in. and 9 sq. in. respectively. If we put sliding caps which just fit inside the respective cylinders and place a 18 lb. weight on each cap, there will be a compression of the material in each cylinder. To find out how this compression depends on the area over which the weight acts, do the following:

a) Find the stress on the material in each of the three cylinders.

b) Express 10 newtons/in^2 in terms of lbs/in^2

c) Find the strain for the material in each of the three cylinders.

d) How far is the material depressed for each cylinder?

e) What does this thought experiment suggest with regard to distribution of forces during an automobile accident in order to prevent local areas of severe injury to the body?

Answer

Denote the columns by i), ii), and iii).

a) i) $181\ \text{lb/in}^2 = 18\ \text{lb/in}^2\ ^2$

ii) $6\ \text{lb/in}^2$

iii) $2\ \text{lb/in}^2$

b) $10^6\ \text{nt/m}^2 = 10 \times 1.45 \times 10\ \text{lb/in}^2 = 145\ \text{lb/in}^2$

c) $\sigma = y \in$

i) $\in = 18\ \text{lb/in}/145\ \text{lb/in} = .12$

ii) $\in = .04$

iii) $\in = .013$

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d) i) $d = .12 \times 12 \text{ in} = 1.4 \text{ in}$

ii) .48 in

iii) .16 in

e) One should try to distribute the forces over the body and try to prevent small areas of contact from taking the brunt of the impact.

Stress and Strain

LABORATORY 4
(Hooke's Law)

I. PURPOSE

The purpose of this lab is to study the behavior of materials when they are subjected to stresses.

II. EQUIPMENT

1 dozen rubber bands, 6 of the same size

1 meter stick

1 support from which to hang rubber band

1 set of weights (100 gm. wts. will do) which may be attached to rubber bands

Several small S shaped hooks to attach wts. to rubber bands

Some graph paper

III. PROCEDURE

A. Using 1 rubber band vary the number of weights and record the strain (extension divided by the original length) of the rubber band. (Also remove the weights one by one to check the reversibility of this experiment.) See accompanying picture for the set up.

B. Repeat A using 2 rubber bands.

C. Repeat A using 4 rubber bands.

D. Plot your data on "a number of weights" versus "extension" curve and explain. (You might try to exceed the breaking point for a rubber band and see what the curve looks like close to that point.) For the linear region of this curve the slope is a Young's modulus for the rubberbands. What value do you find? (Use the correct units.)

E. Repeat this experiment with a metal coil spring if available.

IV. QUESTIONS

A. How is force related to stress?

How is pressure related to stress?

B. What body areas can withstand the most stress?

C. Explain the relation between the rubberbands in this experiment and a seatbelt during an accident.

D. Explain the relation between the rubberbands in this experiment and the body of a passenger.

E. In light of this experiment, explain why it is better to wear both a seat belt and a harness rather than just a seat belt when you are riding in an automobile.

V. COMMENTS FOR THE INSTRUCTOR

A. Explain to the student the physical significance of stress. (How, in general, it is a measure of force applied to a general unit of the deformed body, for example per unit area or in this experiment per rubber band).

B. Explain to the student that the basic materials of both a car and a human body obey definite stress strain laws and that they are elastic to varying limits. When the material (on body) is subjected to too much stress, permanent deformation may take place. Seat belts are effectively very strong rubber bands which tie the human body to the car and protect the various parts of the body from stresses which would exceed their breaking points.

List of Symbols

Distance and Position

x or x(t)	Position or position as a function of time
Δx	Change in position
d	Distance traveled by an object

Time

T	Time interval
t	Any time relative to some observer — variable representing time
Δt	Time interval (read change in time)

Speed and Velocity

V_I or V_o	Initial velocity
V_F	Final velocity
V or V(t)	Instantaneous velocity
V_x	x component of velocity
V_y	y component of velocity
V_{AVE}	Average velocity
ΔV	Change in velocity
S	Speed

Accelerations

a_{AVE}	Average acceleration
a or a(t)	Instantaneous acceleration
a_x, a_y	Components of the instantaneous acceleration

Force

F_{AVE}	Average Force
F(t) or F	Applied force at a certain time

Mass, Momentum and Inpulse

M	Mass
p	Momentum
I	Impulse
Δp	Change in momentum (Impulse)

Energy

W	Work
K. E.	Kinetic Energy

Stress and Strain

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P	Pressure

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